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The Acoustic Impedance of a Vibrating Annular Piston Located on a Flat Rigid Baffle Around a Semi-Infinite Circular Rigid Cylinder

Wojciech P. RDZANEK[(1)], Witold J. RDZANEK[(1)], Dawid PIECZONKA[(2)]

[(1)] *Department of Acoustics, Institute of Physics, University of Rzeszów,*
al. Rejtana 16c, 35-310 Rzeszów, Poland; e-mail: wprdzank@univ.rzeszow.pl

[(2)] *Stylem*
36-001 Trzebownisko 614, Poland

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The axisymmetric problem of acoustic impedance of a vibrating annular piston embedded into a flat rigid baffle concentrically around a semi-infinite rigid cylindrical circular baffle has been undertaken in this study. The Helmholtz equation has been solved. The Green's function valid for the zone considered has been used for this purpose. The influence of the semi-infinite cylindrical baffle on the piston's acoustic impedance has been investigated. The acoustic impedance has been presented in both forms: integral and asymptotic, both valid for the steady harmonic vibrations. Additionally, the acoustic impedances of the piston with and without the cylindrical baffle have been compared to one another. In the case without the cylindrical baffle some earlier results have been used.

Keywords: Neumann boundary value problem; annular piston; acoustic impedance; semi-infinite cylindrical circular rigid baffle.

1. Introduction

The problems of scattering of acoustic waves on obstacles are important from the practical point of view, e.g. in the ultrasound imaging, in the exploration of the sea bottom, etc. The analytical methods as well as the numerical ones are widely used (Faran, 1951; Hasheminejad, Alibakhshi, 2006; Martin, 2011; Stanton, 1992). Analyzing the sound radiation by sound sources located in the vicinity of the obstacles is also important and the results obtained are useful in designing the ultrasound transducers, sonars, etc. The sound radiation phenomena of single flat sources and of systems of such sources located on a flat rigid baffle have been thoroughly examined. Mainly, the acoustic pressure and the acoustic impedance of the vibrating pistons, membranes and plates have been focused (Arenas, 2008; Hashimoto, 2001; Kozień, Wiciak, 2009; Kozupa, Wiciak, 2011; Lee, Singh, 2005; Leniowska, 2008; Levine, Leppington, 1988; Merriweather, 1969; Rdzanek, 1992; Rdzanek *et al.*. 2010; 2011; Stepanishen, 1974; Szemela *et al.*, 2011; Thompson, 1971).

So far, the sound radiation made by a vibrating annular piston located on a flat rigid baffle concentrically around a semi-infinite rigid cylindrical circular baffle has not been analyzed. The Green's function presented by Rdzanek, Rdzanek, and Różycka (2007) can be used in the analysis of the problem. This function has been adapted specifically for the sources located on the semi-infinite cylindrical baffle being perpendicular to the flat rigid baffle. However, it has been shown that under some conditions it can also be used for sources placed on the flat rigid baffle. In this study, the problem of the acoustic impedance of a source located on a flat rigid baffle around a semi-infinite cylindrical circular baffle has been undertaken. The solution of this problem is useful when the radiated acoustic wavelengths are much smaller than the cylindrical baffle radius. Moreover, the influence of an actual cylindrical baffle of finite height on the generated sound field will be similar to the influence of the baffle of semi-infinite height if the actual height is big enough as compared with the radiated wavelengths. Both kinds of formulations, integral and asymptotic, have been obtained for the acoustics impedance of the piston's harmonic steady vibrations, and both are useful for numerical calculations.

2. Governing equations

2.1. Acoustic pressure

The axisymmetric Neumann boundary value problem has been considered for the following zone

$$\Omega = \{a \le r < \infty, \quad 0 \le z < \infty\}$$

bounded by the infinite flat rigid baffle for $z = 0$, and by the semi-infinite cylindrical circular rigid baffle of radius a arranged perpendicularly to the flat baffle (cf. Fig. 1). A vibrating annular piston has been located on the flat baffle concentrically around the cylindrical one. The harmonic time dependence $\mathbf{v}(r, z, t) = \mathbf{v}(r, z) \exp(-\mathrm{i}\omega t)$ has been used for the vibration velocity of the piston as well as of the acoustic particles where ω is the circular frequency and $\mathrm{i}^2 = -1$. It has been assumed that the wave processes are steady in time t which allows conducting any further considerations in the amplitude form valid for a single fixed circular frequency ω. It has also been assumed that the processes are axisymmetric which is possible when the axial symmetry is preserved according to the cylindrical baffle and to the acoustic particle velocity distribution around the baffle. The zone Ω has been filled with the perfectly elastic light fluid, e.g. air. The following Neumann boundary conditions

$$\left.\frac{\partial}{\partial r} G(r, z|r_0, z_0)\right|_{r_0=a} = 0, \qquad \left.\frac{\partial}{\partial z} G(r, z|r_0, z_0)\right|_{z_0=0} = 0 \tag{1}$$

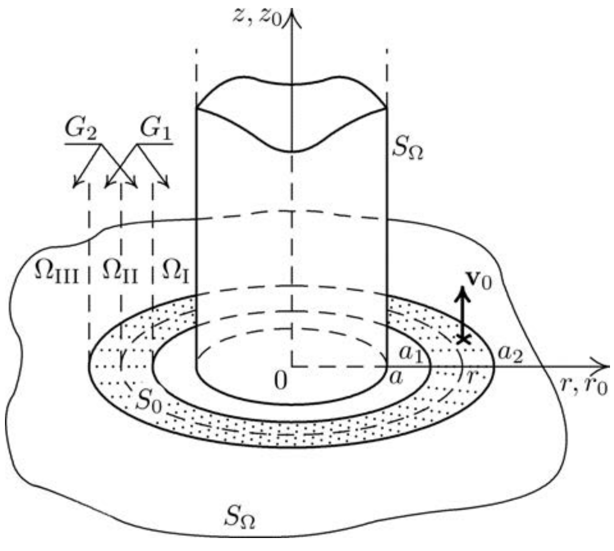

Fig. 1. A vibrating annular piston located on the flat rigid baffle concentrically around a semi-infinite cylindrical circular rigid baffle.

are satisfied at the boundary S_Ω of the zone Ω which, together with the Sommerfeld radiation condition (Rubinowicz, 1971), leads to the Green's function in the following two spectral forms

$$\begin{aligned} G_1(r, z|r_0, z_0) = \frac{\mathrm{i}k}{2\pi} \fint_0^\infty \Bigg[& J_0(kr\sqrt{1-u^2}) \\ & - \frac{J_1(ka\sqrt{1-u^2})}{H_1^{(1)}(ka\sqrt{1-u^2})} H_0^{(1)}(kr\sqrt{1-u^2}) \Bigg] \\ & \cdot H_0^{(1)}(kr_0\sqrt{1-u^2}) \cos(kzu) \cos(kz_0u)\, \mathrm{d}u, \end{aligned}$$

$$\begin{aligned} G_2(r, z|r_0, z_0) = \frac{\mathrm{i}k}{2\pi} \fint_0^\infty \Bigg[& J_0(kr_0\sqrt{1-u^2}) \\ & - \frac{J_1(ka\sqrt{1-u^2})}{H_1^{(1)}(ka\sqrt{1-u^2})} H_0^{(1)}(kr_0\sqrt{1-u^2}) \Bigg] \\ & \cdot H_0^{(1)}(kr\sqrt{1-u^2}) \cos(kzu) \cos(kz_0u)\, \mathrm{d}u \end{aligned} \tag{2}$$

for two sub-zones $\{a \le r \le r_0 < \infty,\ 0 \le z,\ z_0 < \infty\}$ and $\{a \le r_0 \le r < \infty,\ 0 \le z,\ z_0 < \infty\}$ of the zone Ω, respectively, where (r, z) and (r_0, z_0) are the coordinates of the observation and the source points, J_ν, Y_ν, $H_\nu^{(1)}$ are the Bessel function, the Neumann function and the Hankel function of the first kind, all of the order $\nu = 0, 1$, and $\fint$ denotes the principal value. The Green's function in Eqs. (2), which has been presented earlier by Rdzanek, Rdzanek, and Różycka (2007), is the solution of the following non-homogeneous axisymmetric Helmholtz equation

$$(\Delta + k^2)\, G(r, z|r_0, z_0) = -\frac{1}{r}\, \delta(r - r_0)\, \delta(z - z_0) \tag{3}$$

and satisfies the Neumann boundary conditions in Eqs. (1) whereas the integration contour has been presented in Fig. 2.

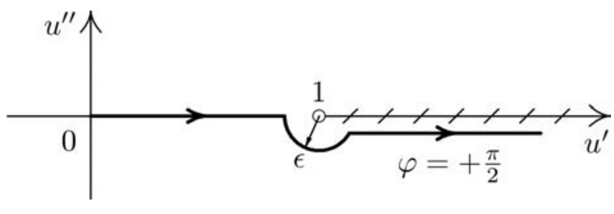

Fig. 2. The integration contour on the complex variable plane $u = u' + \mathrm{i}u'' = |u| \exp(\mathrm{i}\varphi)$ for the Green's function in Eqs. (2).

Further, a vibrating surface sound source has been located on the boundary S_Ω for $z_0 = +0$. In this case, the above mentioned Green's function can be used to calculate the acoustic pressure amplitude by applying the integral Helmholtz-Huygens equation. The acoustic pressure has been formulated as (Morse, Ingard, 1968)

$$p(r,z) = -\mathrm{i}k\varrho_0 c \int\limits_{S_0} v_n(r_0)\, G(r,z|r_0,+0)\,\mathrm{d}S_0, \tag{4}$$

where $v_n(r_0) = \mathbf{n}\cdot\mathbf{v}(r_0)$ is the normal component of the source vibration velocity $\mathbf{v}(r_0)$, $\mathbf{n}$ is the unit vector normal to the source's surface S_0 and $\mathrm{d}S_0$ is the element of this surface. It has been assumed that the source of the fluid disturbance is a vibrating axisymmetric annular piston of its internal and external radii a_1 and a_2, respectively. The normal vibration velocity amplitude of the piston is $v_0 = |v_0|\exp(\mathrm{i}\varphi_0)$ where $|v_0|$ is the modulus and φ_0 is the initial phase. Additionally, it has been assumed that the amplitude is fixed along the entire surface of the vibrating piston, i.e. $|v_0| = \mathrm{const}$ and $\varphi_0 = \mathrm{const}$ whereas on the grounds of Huygens's principle it has been assumed that $v_n(r_0) \equiv v_0$ for $a_1 \le r_0 \le a_2$ and $z_0 = +0$ (Morse, Ingard, 1968). The piston is located on the flat rigid baffle for $z_0 = +0$, concentrically around the cylindrical circular baffle. The case under consideration requires that the acoustic pressure amplitude be formulated separately for the following three sub-zones

$$\Omega_{\mathrm{I}} = \{a \le r \le a_1,\ 0 \le z\},$$

$$\Omega_{\mathrm{II}} = \{a_1 \le r \le a_2,\ 0 \le z\},$$

$$\Omega_{\mathrm{III}} = \{a_2 \le r < \infty,\ 0 \le z\}$$

of the zone Ω (the sub-zones have been separated with the dashed vertical lines in Fig. 1), and the acoustic pressure consecutively assumes the form of

$$\begin{aligned}
p_{\mathrm{I}}(r,z) &= -\,2\pi\mathrm{i}\,k\varrho_0 c\,v_0 \int\limits_{a_1}^{a_2} G_1(r,z|r_0,0)\,r_0\,\mathrm{d}r_0 \qquad \text{for } (r,z)\in\Omega_{\mathrm{I}},\\
p_{\mathrm{II}}(r,z) &= -\,2\pi\mathrm{i}\,k\varrho_0 c\,v_0 \int\limits_{a_1}^{r} G_2(r,z|r_0,0)\,r_0\,\mathrm{d}r_0 \\
&\quad -\,2\pi\mathrm{i}\,k\varrho_0 c\,v_0 \int\limits_{r}^{a_2} G_1(r,z|r_0,0)\,r_0\,\mathrm{d}r_0 \qquad \text{for } (r,z)\in\Omega_{\mathrm{II}},\\
p_{\mathrm{III}}(r,z) &= -\,2\pi\mathrm{i}\,k\varrho_0 c\,v_0 \int\limits_{a_1}^{a_2} G_2(r,z|r_0,0)\,r_0\,\mathrm{d}r_0 \qquad \text{for } (r,z)\in\Omega_{\mathrm{III}},
\end{aligned} \tag{5}$$

where $S_0 = \pi a_1^2\,(s^2-1)$ is the annular piston's area, and $s = a_2/a_1 \in (1,\infty)$ is the quotient of its both radii, external and internal, respectively.

The acoustic pressure amplitude in the sub-zone Ω_{II} for a given radial coordinate r has been presented as a superposition of the two acoustic pressure amplitudes as suggested by Rdzanek, Rdzanek, and Różycka (2007). Both amplitudes have been obtained using the Green's function from Eqs. (2). One of them has been obtained using Eq. $(2)_1$ for $r \le r_0$ and the other one using Eq. $(2)_2$ for $r_0 \le r$.

The acoustic pressure amplitude has been obtained by solving the following homogeneous Helmholtz equation

$$(\Delta + k^2)\,p(r,z) = 0 \tag{6}$$

within the entire zone Ω. Further, the Green's function has been inserted to the acoustic pressure amplitude, yielding

$$\begin{aligned}
p_{\mathrm{I}}(r,z) &= \varrho_0 c v_0 k a_1 \fint_0^{\infty} \Bigg[J_0(kr\sqrt{1-u^2}) - \frac{J_1(s_1 k a_1\sqrt{1-u^2})}{H_1^{(1)}(s_1 k a_1\sqrt{1-u^2})} H_0^{(1)}(kr\sqrt{1-u^2}) \Bigg] \\
&\quad \cdot \widehat{H}_1^{(1)}(s, ka_1\sqrt{1-u^2})\,\frac{\cos(kzu)}{\sqrt{1-u^2}}\,\mathrm{d}u,\\
p_{\mathrm{II}}(r,z) &= i\varrho_0 c\,v_0 \sin(kz) + \varrho_0 c v_0 k a_1 \\
&\quad \cdot \fint_0^{\infty} \Bigg\{ sH_1^{(1)}(ska_1\sqrt{1-u^2}) J_0(kr\sqrt{1-u^2}) \\
&\quad - \Bigg[J_1(ka_1\sqrt{1-u^2}) + \frac{J_1(s_1 k a_1\sqrt{1-u^2})}{H_1^{(1)}(s_1 k a_1\sqrt{1-u^2})} \widehat{H}_1^{(1)}(s, ka_1\sqrt{1-u^2}) \Bigg] \\
&\quad \cdot H_0^{(1)}(kr\sqrt{1-u^2}) \Bigg\} \frac{\cos(kzu)}{\sqrt{1-u^2}}\,\mathrm{d}u,\\
p_{\mathrm{III}}(r,z) &= \varrho_0 c v_0 k a_1 \fint_0^{\infty} \Bigg[\widehat{J}_1(s, ka_1\sqrt{1-u^2}) - \frac{J_1(s_1 k a_1\sqrt{1-u^2})}{H_1^{(1)}(s_1 k a_1\sqrt{1-u^2})} \widehat{H}_1^{(1)}(s, ka_1\sqrt{1-u^2}) \Bigg] \\
&\quad \cdot H_0^{(1)}(kr\sqrt{1-u^2})\frac{\cos(kzu)}{\sqrt{1-u^2}}\,\mathrm{d}u,
\end{aligned} \tag{7}$$

where $s_1 = a/a_1 \in [0,1]$ is the quotient of the cylindrical circular baffle's radius and the annular piston's internal radius, and

$$\begin{aligned}
\widehat{J}_1(s,u) &= sJ_1(su) - J_1(u),\\
\widehat{H}_1^{(1)}(s,u) &= sH_1^{(1)}(su) - H_1^{(1)}(u).
\end{aligned} \tag{8}$$

The first term in Eq. $(7)_2$ has been obtained using the following Wronskian (Watson, 1944) and the

Hilbert transform of the cosine function (Bracewell, 1999)

$$\begin{gathered} J_1(u)H_0^{(1)}(u) - J_0(u)H_1^{(1)}(u) = \frac{2\mathrm{i}}{\pi u}, \\ \frac{2}{\pi}\,\fint_0^\infty \frac{\cos(kzu)\,\mathrm{d}u}{1-u^2} = \sin(kz), \end{gathered} \tag{9}$$

for $0 \leq z$ and $0 < k$. It can be proved that the continuity conditions of the acoustic pressure amplitude and of its gradient are satisfied at the boundaries of the three considered sub-zones, i.e. at $r = a_1$ and $r = a_2$ for $0 < z < \infty$

$$\begin{gathered} p_{\mathrm{I}}(a_1, z) = p_{\mathrm{II}}(a_1, z), \\ \nabla p_{\mathrm{I}}(r, z)\Big|_{r=a_1} = \nabla p_{\mathrm{II}}(r, z)\Big|_{r=a_1}, \\ p_{\mathrm{II}}(a_2, z) = p_{\mathrm{III}}(a_2, z), \\ \nabla p_{\mathrm{II}}(r, z)\Big|_{r=a_2} = \nabla p_{\mathrm{III}}(r, z)\Big|_{r=a_2}. \end{gathered} \tag{10}$$

The Hilbert transform of the sine function (Bracewell, 1999)

$$\frac{2}{\pi}\,\fint_0^\infty \frac{\sin(kzu)\,u\,\mathrm{d}u}{1-u^2} = -\cos(kz), \tag{11}$$

for $0 < z$ and $0 < k$ has been used for this purpose. Obviously, satisfying the continuity conditions from Eqs. (10) is necessary to assure that the acoustic pressure $p(r, z)$ from Eqs. (7) can be the solution of the Helmholtz equation (6).

2.2. Acoustic power in the integral form

The time averaged acoustic power is equal to the total flux of acoustic intensity $p\mathbf{v}^*$ passing through a surface enclosing the source of fluid disturbance (Morse, Ingard, 1968). The impedance approach has been used while calculating the acoustic power, i.e. the enclosing surface S has been selected near the surface S_0 of a vibrating piston for $z = +0$. The acoustic power Π and the reference power $\Pi^{(\infty)}$ assume the following forms

$$\begin{aligned} \Pi &= \frac{1}{2}\int_S p v_n^* \mathrm{d}S = \pi v_0^* \int_{a_1}^{a_2} p_{\mathrm{II}}(r, +0)\, r\,\mathrm{d}r, \\ \Pi^{(\infty)} &= \lim_{k\to\infty} \Pi_0 = \frac{1}{2}\,\varrho_0 c \int_S |v_n|^2 \mathrm{d}S \\ &= \frac{1}{2}\,\varrho_0 c |v_0|^2 S_0, \end{aligned} \tag{12}$$

where $pv_n^*\,\mathrm{d}S = p\,(\mathbf{v}^* \cdot \mathrm{d}\mathbf{S}) = p\,(\mathbf{v}^* \cdot \mathbf{n})\,\mathrm{d}S$, $v_n^* \equiv v_0^*$, * denotes the conjugate value, $\mathbf{n}$ is the unit vector normal to the surface S, $\mathrm{d}S$ is the element of this surface, $p_{\mathrm{II}}(r, +0)$ is the acoustic pressure amplitude from Eq. $(7)_2$ valid for $a_1 \leq r \leq a_2$ and $z = +0$. The surface S is equivalent to the surface S_0 over which the integration has been performed in Eq. (4) while calculating the acoustic pressure amplitude. The normalized acoustic impedance has been formulated as follows

$$\begin{aligned} \zeta &= \frac{\Pi}{\Pi^{(\infty)}} = \frac{2}{s^2-1}\fint_0^\infty \overline{H}(s, s_1, ka_1\sqrt{1-u^2})\frac{\mathrm{d}u}{1-u^2} \\ &= \frac{2}{s^2-1}\int_0^1 \overline{H}(s, s_1, ka_1\sqrt{1-u^2})\frac{\mathrm{d}u}{1-u^2} \\ &\quad + \frac{2}{s^2-1}\int_1^\infty \overline{K}(s, s_1, ka_1\sqrt{u^2-1})\frac{\mathrm{d}u}{1-u^2}, \end{aligned} \tag{13}$$

where

$$\begin{aligned} \overline{H}(s,s_1,u) &= sH_1^{(1)}(su)\widehat{J}_1(s,u) - \widehat{H}_1^{(1)}(s,u) \\ &\quad \cdot \left[J_1(u) + \frac{J_1(s_1u)}{H_1^{(1)}(s_1u)}\widehat{H}_1^{(1)}(s,u)\right], \\ \overline{K}(s,s_1,u) &= -\frac{2i}{\pi}\Big\{sK_1(su)\widehat{I}_1(s,u) \\ &\quad - \widehat{K}_1(s,u)\left[I_1(u) + \frac{I_1(s_1u)}{K_1(s_1u)}\widehat{K}_1(s,u)\right]\Big\} \end{aligned} \tag{14}$$

and

$$\begin{gathered} \widehat{I}_1(s,u) = sI_1(su) - I_1(u), \\ \widehat{K}_1(s,u) = sK_1(su) - K_1(u). \end{gathered} \tag{15}$$

The last form in Eq. (13) is useful for numerical calculation since the singular points appear only at the boundaries of integration interval.

In the limiting case when $s_1 \to 1$, Eqs. $(14)_1$ and $(14)_2$ simplify to

$$\begin{aligned} \lim_{s_1\to1} \overline{H}(s,s_1,u) &= s^2\,\frac{H_1^{(1)}(su)}{H_1^{(1)}(u)} \\ &\quad \cdot \left[J_1(su)H_1^{(1)}(u) - J_1(u)H_1^{(1)}(su)\right], \\ \lim_{s_1\to1} \overline{K}(s,s_1,u) &= -\frac{2i}{\pi}\,s^2\,\frac{K_1(su)}{K_1(u)} \\ &\quad \cdot \left[I_1(su)K_1(u) - I_1(u)K_1(su)\right]. \end{aligned} \tag{16}$$

Further, the following substitution has been performed $\sqrt{1-u^2} = x$ yielding

$$\begin{aligned}
\zeta &= \zeta_1 + \zeta_2 \equiv \theta - \mathrm{i}\chi,\\
\zeta_1 &= \frac{2}{s^2-1} \int\limits_{L_1} \frac{F_1(s, ka_1\sqrt{1-u^2})\,\mathrm{d}u}{1-u^2}\\
&= \frac{2}{s^2-1} \int\limits_{L_2} \frac{F_1(s, ka_1 x)\,\mathrm{d}x}{x\sqrt{1-x^2}},\\
\zeta_2 &= \frac{2}{s^2-1} \int\limits_{L_1} \frac{F_2(s, s_1, ka_1\sqrt{1-u^2})\,\mathrm{d}u}{1-u^2}\\
&= \frac{2}{s^2-1} \int\limits_{L_2} \frac{F_2(s, s_1, ka_1 x)\,\mathrm{d}x}{x\sqrt{1-x^2}},
\end{aligned} \tag{17}$$

where $\theta = \mathrm{Re}\,\zeta$ is the normalized radiation resistance, $\chi = -\,\mathrm{Im}\,\zeta$ is the normalized radiation reactance, and

$$\begin{aligned}
F_1(s,u) &= sH_1^{(1)}(su)\widehat{J}_1(s,u) - J_1(u)\widehat{H}_1^{(1)}(s,u),\\
F_2(s,s_1,u) &= -\frac{J_1(s_1 u)}{H_1^{(1)}(s_1 u)} \left[\widehat{H}_1^{(1)}(s,u)\right]^2
\end{aligned} \tag{18}$$

with the integration contours $L_1 = \overrightarrow{AB} + \overrightarrow{CD}$ and $L_2 = \overrightarrow{AB} + \overrightarrow{CD}$ presented in Figs. 3a and 3b, respectively. The integrals in Eqs. (17) are the principal values.

The component ζ_1 represents the acoustic radiation impedance of a vibrating annular piston located on a flat rigid baffle without the cylindrical circular baffle whereas the component ζ_2 represents the correction for the presence of the cylindrical circular baffle since for $s_1 \to 0$ (which is equivalent with $a \to 0$) the cylindrical baffle vanishes completely, and

$$\lim_{s_1\to 0} \frac{J_1(s_1 u)}{H_1^{(1)}(s_1 u)} = \lim_{s_1\to 0} \frac{I_1(s_1 u)}{K_1(s_1 u)} = 0$$

for u on the contours L_1 and L_2 or within their interiors.

The integrals in Eqs. (17) are useless for numerical calculations. Therefore, they have been formulated as follow

$$\begin{aligned}
\zeta_1 &= 1 - \frac{2\mathrm{i}}{s^2-1} \int\limits_0^\infty \frac{F_1(s, ka_1\sqrt{1+y^2})\,\mathrm{d}y}{1+y^2}\\
&= 1 - \frac{2\mathrm{i}}{s^2-1} \int\limits_1^\infty \frac{F_1(s, ka_1 x)\,\mathrm{d}x}{x\sqrt{x^2-1}},\\
\zeta_2 &= -\frac{2\mathrm{i}}{s^2-1} \int\limits_0^\infty \frac{F_2(s, s_1, ka_1\sqrt{1+y^2})\,\mathrm{d}y}{1+y^2}\\
&= -\frac{2\mathrm{i}}{s^2-1} \int\limits_1^\infty \frac{F_2(s, s_1, ka_1 x)\,\mathrm{d}x}{x\sqrt{x^2-1}}
\end{aligned} \tag{19}$$

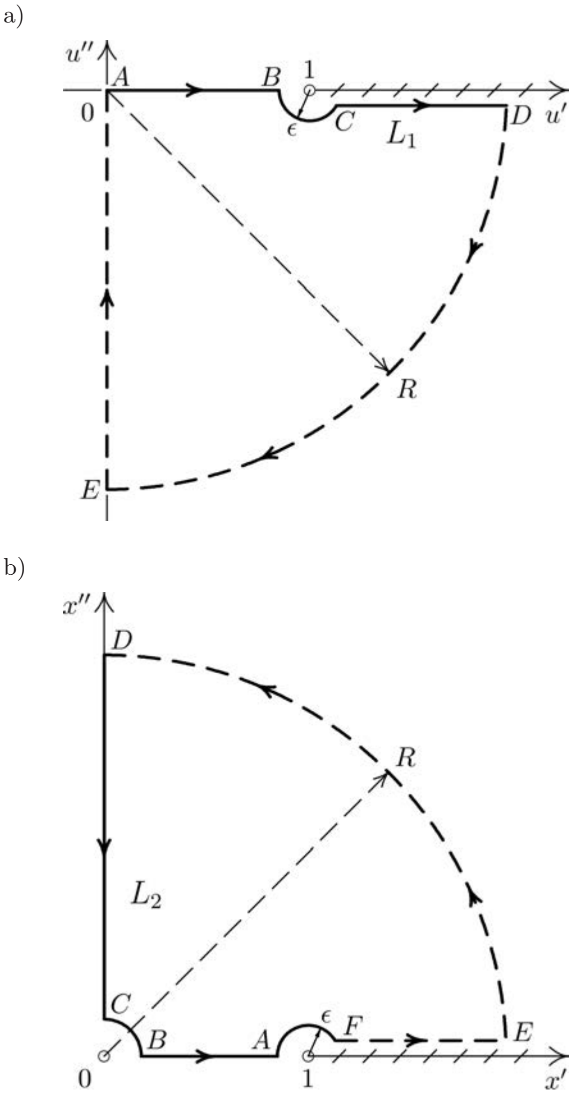

Fig. 3. The integration contours in Eqs. (17) for $r \to \infty$ and $\epsilon \to 0$ on the complex variable planes $u = u' + \mathrm{i}u''$ and $x = x' + \mathrm{i}x''$, respectively. key: solid line – open contours l_1 and l_2, dashed line – complementary parts of the corresponding closed curves.

by using the Cauchy's theorem (cf. Appendix A). The obtained formulations are free of the inconvenient singularities and further they will be used in both analyses: theoretical and numerical.

2.3. Sound radiation without the cylindrical circular baffle

Applying the proper substitution and the Cauchy's theorem it can be proven that the integral formulations $(17)_2$ and $(19)_1$ for ζ_1 are equivalent with those presented earlier by Merroweather (1969); Stepanishen (1974); Thompson (1971) (cf. Appendix A) and represent the acoustic impedance of a vibrating annular piston located on the flat rigid baffle. The analysis of this quantity will be presented in a shortened form in order to show how the literature results can be used to partially express the acoustic impedance of the piston, with the semi-infinite cylindrical baffle added, in the elementary non integral form. The axisymmetric Green's function for the zone $\Omega' = \{0 \leq z,\ z_0 = +0\}$ located above the flat rigid baffle for $z = +0$ has been formulated as follows (Morse, Ingard, 1968)

$$G(r, z|r_0, +0) = \frac{\mathrm{i}k}{2\pi} \int\limits_0^\infty \exp(\mathrm{i}kz\sqrt{1-x^2}) \cdot J_0(krx) J_0(kr_0x) \frac{x\,\mathrm{d}x}{\sqrt{1-x^2}} \tag{20}$$

with the integration contour presented in Fig. 2. This function is the solution of the non homogeneous Helmholtz equation within the zone Ω'. The acoustic pressure amplitude has been formulated as

$$\begin{aligned} p(r, z) &= -2\pi\mathrm{i}\, k\varrho_0 c\, v_0 \int\limits_{a_1}^{a_2} G(r, z|r_0, +0)\, r_0\, \mathrm{d}r_0 \\ &= k^2 \varrho_0 c\, S_0 \int\limits_0^\infty \exp(\mathrm{i}kz\sqrt{1-x^2}) \\ &\quad \cdot J_0(krx) M(x) \frac{x\,\mathrm{d}x}{\sqrt{1-x^2}}, \end{aligned} \tag{21}$$

where $S_0 = \pi a^2 (s^2 - 1)$ is the area of the vibrating annular piston, $s = a_2/a_1$ and

$$\begin{aligned} M(x) &= \frac{v_0}{S_0} \int\limits_{a_1}^{a_2} J_0(kr_0x)\, r_0\, \mathrm{d}r_0 \\ &= \frac{v_0 a_1}{S_0 kx} \Big[sJ_1(ska_1x) - J_1(ka_1x) \Big]. \end{aligned} \tag{22}$$

The acoustic power has been formulated as follows

$$\Pi_1 = \pi k^2 \varrho_0 c\, S_0^2 \int\limits_0^\infty M(x) M^*(x) \frac{x\,\mathrm{d}x}{\sqrt{1-x^2}} \tag{23}$$

according to the definition in Eq. $(12)_1$, the reference power has been already defined in Eq. $(12)_2$ whereas the normalized acoustic impedance has been formulated as

$$\zeta_1 = \frac{2}{s^2-1} \int\limits_0^\infty \Big[sJ_1(ska_1x) - J_1(ka_1x) \Big]^2 \cdot \frac{\mathrm{d}x}{x\sqrt{1-x^2}} \tag{24}$$

according to the definition in Eq. (13). The above integral can expressed as the sum of three integrals. The two of them represent the normalized acoustic impedance of the vibrating circular pistons of radii a_2 and a_1. The exact non integral impedance formulation of such sources

$$\int\limits_0^\infty \frac{2J_1^2(bx)\,\mathrm{d}x}{x\sqrt{1-x^2}} = 1 - \frac{J_1(2b)}{b} - \mathrm{i}\frac{\mathbf{H}_1(2b)}{b} \tag{25}$$

has been presented earlier, e.g. by Morse and Ingard (1968); Thompson (1971), for $0 < b$, where $\mathbf{H}_1$ is the Struve function of the first order, and the integration contour has been presented in Fig. 2. This expression represents the acoustic impedance of a vibrating circular piston of radius b located on the flat rigid infinite baffle. It is worth noticing that an interesting approximation for the Struve function has been proposed by Janssem (2003). This approximation is valid within the entire range of the argument and makes the numerical computations considerably faster. Applying Eq. (25) yields the following, partially non integral, formulation

$$\begin{aligned} \zeta_1 =& \frac{-1}{(s^2-1)\,ka_1} \Big\{ sJ_1(2ska_1) + J_1(2ka_1) \\ & + \mathrm{i}\Big[s\mathbf{H}_1(2ska_1) + \mathbf{H}_1(2ka_1) \Big] \Big\} + \frac{s^2+1}{s^2-1} \\ & - \frac{4s}{s^2-1} \int\limits_0^\infty \frac{J_1(ska_1x) J_1(ka_1x)\,\mathrm{d}x}{x\sqrt{1-x^2}} \end{aligned} \tag{26}$$

for the acoustic impedance from Eq. (24). Unfortunately, the integral appearing in the above formulation can not be formulated in the non integral form since it contains the product of the two Bessel functions of the two different arguments, ska_1x and ka_1x, for $1 < s$. The acoustic impedance approximations valid for small values of the wave parameter ka_1 have been presented earlier by Merriweather (1969); Stepanishen (1974); Thompson (1971) whereas for big values of this parameter the method of contour integral should be used in a similar way as it has been performed in the analysis of sound radiation by a vibrating circular plate (Levine, Leppington, 1988; Rdzanek, 1992, cf. also Appendix 4.). First, the integral has been formulated as

$$\begin{aligned} \frac{4s}{s^2-1} & \int\limits_0^\infty \frac{J_1(ska_1x) J_1(ka_1x)\,\mathrm{d}x}{x\sqrt{1-x^2}} \\ &= \frac{4s}{s^2-1} \int\limits_0^1 \frac{J_1(ska_1x) J_1(ka_1x)\,\mathrm{d}x}{u\sqrt{1-x^2}} \\ &\quad - \frac{4\mathrm{i}s}{s^2-1} \int\limits_1^\infty \frac{J_1(ska_1x) J_1(ka_1x)\,\mathrm{d}x}{x\sqrt{x^2-1}}. \end{aligned} \tag{27}$$

The asymptotic value of the integral $\int\limits_1^\infty$ can be calculated immediately using the stationary phase method.

However, calculating the asymptotic value of the integral $\int_0^1$ requires analyzing the following contour integral

$$\oint_L \frac{F(x)\,\mathrm{d}x}{x\sqrt{1-x^2}} = 0, \tag{28}$$

where L is the closed Jordan curve on the plane of complex variable x (denoted with the solid and dashed lines in Fig. 3b), and the following function

$$F(x) = \frac{4s}{s^2-1} J_1(ka_1x)H_1^{(1)}(ska_1x) \tag{29}$$

is analytic on the curve L and inside it. It is worth noticing that using the Green's function from Eqs. (2) (presented earlier by Rdzanek *et al.*, 2007) yields the function from Eq. (29) directly in such a form that the argument of the Hankel function is equal to or greater than the argument of the Bessel function which is the sufficient convergence condition of the integral in Eq. (28) whereas it was necessary to construct a similar function in the special way to assure the convergence of a contour integral similar as in the studies by Levine and Leppington (1988); Rdzanek (1992). The integral from Eq. (28) can be formulated as

$$\int_0^1 \frac{F(x)\,\mathrm{d}x}{x\sqrt{1-x^2}} - \frac{\pi \mathrm{i}}{2}\operatorname*{Res}_{x=0}\left[\frac{F(x)}{x\sqrt{1-x^2}}\right] + \int_1^\infty \frac{F(x)\,\mathrm{d}x}{-\mathrm{i}x\sqrt{x^2-1}} + \int_\infty^0 \frac{-F(\mathrm{i}y)\,\mathrm{d}y}{y\sqrt{1+y^2}} = 0 \tag{30}$$

given that $\lim\limits_{R\to\infty,\,\epsilon\to 0}\int_{\widehat{EF}} = 0$ in the virtue of the Jordan lemma, $\lim\limits_{\epsilon\to 0}\int_{\widehat{CD}} = 0$ due to vanishing of the residue at point $x=1$, and $\lim\limits_{\epsilon\to 0}\int_{\widehat{AB}}$ is equal to one fourth of the residue at point $x=0$ where the first order pole appears. Taking the real part of Eq. (30), and using the fact that $\lim\limits_{\epsilon\to 0}\operatorname{Re}\int_{\overrightarrow{FA}} = 0$ since $\operatorname{Re}F(\mathrm{i}y) = 0$, yields

$$\begin{aligned}\operatorname{Re}\int_0^1 \frac{F(x)\,\mathrm{d}x}{x\sqrt{1-x^2}} &= \frac{4s}{s^2-1}\int_0^1 \frac{J_1(ska_1x)J_1(ka_1x)\,\mathrm{d}x}{x\sqrt{1-x^2}} \\ &= \operatorname{Re}\left\{\frac{\pi\mathrm{i}}{2}\operatorname*{Res}_{x=0}\left[\frac{F(x)}{x\sqrt{1-x^2}}\right]\right\} \\ &\quad + \int_1^\infty \frac{\operatorname{Im}[F(x)]\,\mathrm{d}x}{x\sqrt{x^2-1}}.\end{aligned} \tag{31}$$

After inserting into Eq. (27) it has been obtained that

$$\frac{4s}{s^2-1}\int_0^\infty \frac{J_1(ska_1x)J_1(ka_1x)\,\mathrm{d}x}{x\sqrt{1-x^2}} = \frac{2}{s^2-1} - \frac{4\mathrm{i}s}{s^2-1}\int_1^\infty \frac{J_1(ka_1x)H_1^{(1)}(ska_1x)\,\mathrm{d}x}{x\sqrt{x^2-1}} \tag{32}$$

which inserted to Eq. (26) results in

$$\begin{aligned}\zeta_1 = 1 &- \frac{1}{(s^2-1)ka_1}\Big\{sJ_1(2ska_1) + J_1(2ka_1) \\ &+ \mathrm{i}\,[s\mathbf{H}_1(2ska_1) + \mathbf{H}_1(2ka_1)]\Big\} \\ &+ \frac{4\mathrm{i}s}{s^2-1}\int_1^\infty \frac{J_1(ka_1x)H_1^{(1)}(ska_1x)\,\mathrm{d}x}{x\sqrt{x^2-1}}.\end{aligned} \tag{33}$$

This equation has been formulated partially in an elementary form and partially in an integral form, and is it useful for numerical calculations of the normalized acoustic impedance of a vibrating annular piston located on a flat rigid baffle. The asymptotic formula for the integral in the above equation has been obtained using the following asymptotic expansion (cf. Eq. (48))

$$\begin{aligned}4\mathrm{i}sJ_1(ka_1x)H_1^{(1)}(ska_1x) = &-\frac{4\sqrt{s}}{\pi ka_1x} \cdot \Big\{\exp[\mathrm{i}(s+1)ka_1x] - \mathrm{i}\exp[\mathrm{i}(s-1)ka_1x]\Big\} \\ &+ \frac{3\sqrt{s}}{2\pi(ka_1)^2x^2}\Big\{\exp[\mathrm{i}(s-1)ka_1x] \\ &- \mathrm{i}\exp[\mathrm{i}(s+1)ka_1x]\Big\} + \mathcal{O}[(ka_1x)^{-3}]\end{aligned} \tag{34}$$

which has been inserted to the definite integral in Eq. (33). Further, the asymptotic stationary phase method has been used (cf. Appendix B), and the following substitution $x^2 = 1 + w^2$ has been applied, giving

$$\begin{aligned}\zeta_1 = 1 &- \frac{1}{(s^2-1)ka_1}\Big\{sJ_1(2ska_1) + J_1(2ka_1) \\ &+ \mathrm{i}\,[s\mathbf{H}_1(2ska_1) + \mathbf{H}_1(2ka_1)]\Big\} \\ &- \frac{2\sqrt{2s}}{(s^2-1)ka_1\sqrt{\pi}} \\ &\cdot \Bigg\{\left(1+\frac{3\mathrm{i}}{8ka_1}\right)\frac{\exp\{\mathrm{i}[(s+1)ka_1+\pi/4]\}}{\sqrt{(s+1)ka_1}} \\ &- \left(\mathrm{i}+\frac{3}{8ka_1}\right)\frac{\exp\{\mathrm{i}[(s-1)ka_1+\pi/4]\}}{\sqrt{(s-1)ka_1}} \\ &+ \mathcal{O}\left[(ka_1)^{-1}\right]\Bigg\}\end{aligned} \tag{35}$$

which represents the asymptotic formulation of the acoustic impedance of a vibrating annular piston located on the flat rigid baffle. It is valid for big values of the wave parameter $1 \ll ka_1$. It is worth noticing that this formulation has been expressed partially in the exact form, using the Bessel and Struve functions, both of the first order, and partially in the asymptotic form.

2.4. Correction for the cylindrical circular baffle

The cylindrical circular baffle become influencing the sound radiation when $0 < a$. In this case, the correction ζ_2 from Eqs. $(17)_3$ and $(19)_2$ should be included. It can be formulated as follows

$$\zeta_2 = \frac{2\mathrm{i}}{s^2-1} \int\limits_1^\infty \frac{J_1(s_1 ka_1 x)}{H_1^{(1)}(s_1 ka_1 x)} \cdot \left[\widehat{H}_1^{(1)}(s, ka_1 x)\right]^2 \frac{\mathrm{d}x}{x\sqrt{x^2-1}}. \tag{36}$$

After expanding according to Eq. $(8)_2$, the above integrand contains the products of two cylindrical functions of two different arguments (cf. Eqs. $(8)_2$ and $(18)_2$), and therefore it is not possible to find the exact formulation for the corresponding integral. The integral formulation for the correction as well as the asymptotic formulation have not been presented earlier. In order to find the asymptotic one, the same procedure is to be conducted as in Subsec. 2.3. with the exception that now the residue at $x = 0$ is equal to zero and the asymptotic formulation of the definite integral has been calculated using the following asymptotic expansion (cf. Eq. (48))

$$\begin{aligned}
2\mathrm{i}\,\frac{J_1(s_1 ka_1 x)}{H_1^{(1)}(s_1 ka_1 x)} \left[\widehat{H}_1^{(1)}(s, ka_1 x)\right]^2 \approx \frac{2}{\pi ka_1 x} \\
\cdot \Big(-s\exp(2\mathrm{i}ska_1x) + 2\sqrt{s}\exp[\mathrm{i}(s+1)ka_1x] \\
- \exp(2\mathrm{i}ka_1x) + \mathrm{i}s\exp[2\mathrm{i}(s-s_1)ka_1x] - 2\mathrm{i}\sqrt{s} \\
\cdot \exp[\mathrm{i}(s+1-2s_1)ka_1x] + \mathrm{i}\exp[2\mathrm{i}(1-s_1)ka_1x]\Big) \\
+ \frac{6\mathrm{i}}{8\pi(ka_1x)^2}\Big(- s\exp(2\mathrm{i}ska_1x) \\
+ 2\sqrt{s}\,\exp[\mathrm{i}(s+1)ka_1x] - \exp(2\mathrm{i}ka_1x) \\
- \mathrm{i}s\exp[2\mathrm{i}(s-s_1)ka_1x] \\
+ 2\mathrm{i}\sqrt{s}\,\exp[\mathrm{i}(s+1-2s_1)ka_1x] \\
- \mathrm{i}\exp(2\mathrm{i}(1-s_1)ka_1x)\Big).
\end{aligned} \tag{37}$$

Further, the zero expansion term of the stationary phase method has been used giving

$$\begin{aligned}
\zeta_2 = &\frac{\sqrt{2}}{(s^2-1)ka_1\sqrt{\pi}}\Bigg\{ - \left(1 + \frac{3\mathrm{i}}{8ka_1}\right) \\
&\cdot \left(s\,\frac{\exp[\mathrm{i}(2ska_1+\pi/4)]}{\sqrt{2ska_1}} - 2\sqrt{s}\right. \\
&\left.\cdot \frac{\exp\{\mathrm{i}[(s+1)ka_1+\pi/4]\}}{\sqrt{(s+1)ka_1}} + \frac{\exp[\mathrm{i}(2ka_1+\pi/4)]}{\sqrt{2ka_1}}\right) \\
&+ \left(\mathrm{i} + \frac{3}{8ka_1}\right)\left(s\,\frac{\exp[\mathrm{i}(2(s-s_1)ka_1+\pi/4)]}{\sqrt{2(s-s_1)ka_1}}\right. \\
&- 2\sqrt{s}\,\frac{\exp\{\mathrm{i}[(s+1-2s_1)ka_1+\pi/4]\}}{\sqrt{(s+1-2s_1)ka_1}} \\
&\left.+ \frac{\exp[\mathrm{i}(2(1-s_1)ka_1+\pi/4)]}{\sqrt{2(1-s_1)ka_1}}\right)\Bigg\} \\
&+ \mathcal{O}\left[(ka_1)^{-1}\right]
\end{aligned} \tag{38}$$

which represents the asymptotic formulation of the acoustic impedance correction of a vibrating annular piston valid for big values of the wave parameter $1 \ll ka_1$, associated with the influence of the cylindrical circular baffle on the generated acoustic field.

The cylindrical baffle vanishes when $s_1 \to 0$ ($a \to 0$ for $0 < a_1 = \mathrm{const}$). The acoustic impedance considered in Subsec. 2.3. concerns the sound radiation of a vibrating annular piston located on the flat rigid baffle. Although the asymptotic formulation presented in this section has been obtained for the entire interval $0 < a < a_1$, it introduces a considerable error for $a \sim 0$ and should not be used in this case. From a practical point of view, the influence of the cylindrical baffle of vanishing radius is negligible and in this case, the formulations from Subsec. 2.3. are useful.

There is an even more interesting case for $s_1 \to 1$ ($a \to a_1$), i.e. when the radius of the cylindrical baffle achieves the maximum value equal to the internal radius of the vibrating annular piston and the influence of the baffle on the sound field becomes the biggest. Consequently, the asymptotic formulations for the acoustic impedance in Eq. (38) also introduce a considerable error and should not be used. From a practical point of view, it is useful to assume that $a \sim a_1$ and to conduct the analysis of the asymptotic formulation separately. For this purpose, the following limit

$$\lim_{s_1\to 1} F_2(s, s_1, u) = -s^2\frac{J_1(u)}{H_1^{(1)}(u)}\left[H_1^{(1)}(su)\right]^2 + 2sJ_1(u)H_1^{(1)}(su) - J_1(u)H_1^{(1)}(u) \tag{39}$$

has been inserted to Eq. $(19)_2$ which results in

$$\lim_{s_1\to 1}\zeta_2 = \frac{1}{(s^2-1)ka_1}\left[J_1(2ka_1)+\mathrm{i}\mathbf{H}_1(2ka_1)\right] - \frac{4\mathrm{i}s}{s^2-1}\int_1^\infty \frac{J_1(ka_1x)H_1^{(1)}(ska_1x)\,\mathrm{d}x}{x\sqrt{x^2-1}} + \frac{2\mathrm{i}s^2}{s^2-1}\int_1^\infty \frac{J_1(ka_1x)}{H_1^{(1)}(ka_1x)} \cdot \left[H_1^{(1)}(ska_1x)\right]^2 \frac{\mathrm{d}x}{x\sqrt{x^2-1}} \tag{40}$$

after using Eq. (25). Further, this equation added to Eq. (33) yields

$$\lim_{s_1\to 1}\zeta = 1 - \frac{s}{(s^2-1)ka_1}\left[J_1(2ska_1)+\mathrm{i}\mathbf{H}_1(2ska_1)\right] + \frac{2\mathrm{i}s^2}{s^2-1}\int_1^\infty \frac{J_1(ka_1x)}{H_1^{(1)}(ka_1x)} \cdot \left[H_1^{(1)}(ska_1x)\right]^2 \frac{\mathrm{d}x}{x\sqrt{x^2-1}}. \tag{41}$$

The above formulation is useful for numerical calculations. The asymptotic formulation for the integral has been obtained using the following asymptotic expansion (cf. (48))

$$2\mathrm{i}s^2\frac{J_1(u)}{H_1^{(1)}(u)}\left[H_1^{(1)}(su)\right]^2 = \frac{2s}{\pi u}\Big\{-\exp(2\mathrm{i}su)+\mathrm{i}\exp[2\mathrm{i}(s-1)u]\Big\} + \frac{3s}{4\pi u^2}\Big\{\exp[2\mathrm{i}(s-1)u]-\mathrm{i}\exp(2\mathrm{i}su)\Big\} + \mathcal{O}(u^{-3}). \tag{42}$$

This formulation has been inserted to Eq. (41) and the stationary phase method has been used (cf. Appendix 4.). For this purpose, the following substitution has been applied $x^2 = 1 + w^2$ giving

$$\lim_{s_1\to 1}\zeta = 1 - \frac{s}{(s^2-1)ka_1}\Big[J_1(2ska_1) + \mathrm{i}\mathbf{H}_1(2ska_1)\Big] + \frac{s\sqrt{2}}{(s^2-1)ka_1\sqrt{\pi}} \cdot \Bigg\{-\left(1+\frac{3\mathrm{i}}{8ka_1}\right)\frac{\exp\{\mathrm{i}(2ska_1+\pi/4)\}}{\sqrt{2ska_1}} + \left(\mathrm{i}+\frac{3}{8ka_1}\right)\frac{\exp\{\mathrm{i}[2(s-1)ka_1+\pi/4]\}}{\sqrt{2(s-1)ka_1}} + \mathcal{O}\left[(ka_1)^{-1}\right]\Bigg\} \tag{43}$$

which represents the asymptotic formulation of the acoustic impedance of a vibrating annular piston located in a flat rigid baffle concentrically around a cylindrical circular baffle in the specific case when $s_1 \to 1$ and for big values of the wave parameter $1 \ll ka_1$. It is worth noticing that due to applying the well-known formulation for the acoustic impedance of a vibrating circular piston located in the flat rigid baffle, the above expression has been obtained partially in its non integral exact form.

3. Numerical analysis

As mentioned at the beginning of this study, both integral and asymptotic formulations are useful for numerical calculations. The integral ones have this feature that they are valid within the entire range of acoustic wavenumber k, and the only error results from the numerical integration procedures used. However, it is obvious that numerical integration is time consuming and therefore the asymptotic formulations have also been considered.

Figure 4 shows the normalized acoustic impedance, i.e. the resistance θ (Fig. 4a) and the reactance χ (Fig. 4b), and the absolute error $|\Delta\zeta|$ (Fig. 4c). The integral formulations have been used to investigate the influence of the piston's geometric parameter s on the acoustic impedance within a wide band of k. It has been arbitrarily assumed that $S = 0.5$ m^2 and $s_1 = 0.5$ for the three different values of s for comparison. The behavior of the piston's acoustic impedance for all three values of s is similar to the case of no cylindrical circular baffle, i.e. the number of curve oscillations per interval k increases with a decrease in s. Generally, the number of oscillations is slightly smaller than in the case of no cylindrical baffle due to scattering on the cylinder (Merriweather, 1969; Thompson, 1971) which applies to both – resistance and reactance. The absolute value of the error of asymptotic formulations has been estimated as follows. First, both integral and asymptotic values have been calculated numerically as the functions of k. The integral value has been used as the reference and the absolute difference of both values has been regarded as the error order estimation. Only the modulus error estimation $|\Delta\zeta|$ has been presented, since $|\Delta\theta|$, $|\Delta\chi| \le |\Delta\zeta|$ and the resistance and reactance errors assume similar values to one another within the entire considered interval of acoustic wavenumber k. Therefore, the quantity $|\Delta\zeta|$ is useful for estimating the investigated asymptotic formulations errors.

The error assumes the biggest values for the lowest analyzed value of $s = 1.2$ and, generally, decreases with an increase in s. The two dotted horizontal lines help reading the values of k at which the error falls below 10^{-1} and below 10^{-2}, respectively. The most desirable values of s, from practical point of view, are those $s \to 2$ which can be argued as follows. For small values $s \to 1$, the vibrating piston becomes a very narrow annulus of a vanishing active surface. In this case, the active acoustic power is negligibly small and con-

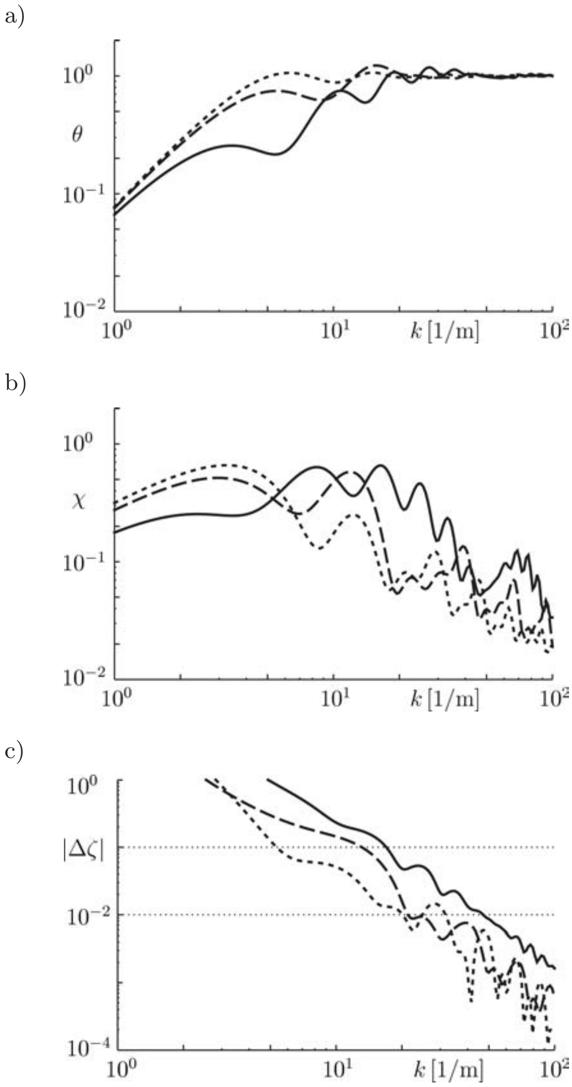

Fig. 4. The normalized acoustic impedance ζ as a function of the acoustic wavenumber k for $s_1 = 0.5$, $S = 0.5\mathrm{m}^2$: a) resistance θ, b) reactance χ, c) error $|\Delta\zeta|$. Key: solid line $s = 1.2$, dashed line $s = 2$, dotted line $s = 5$.

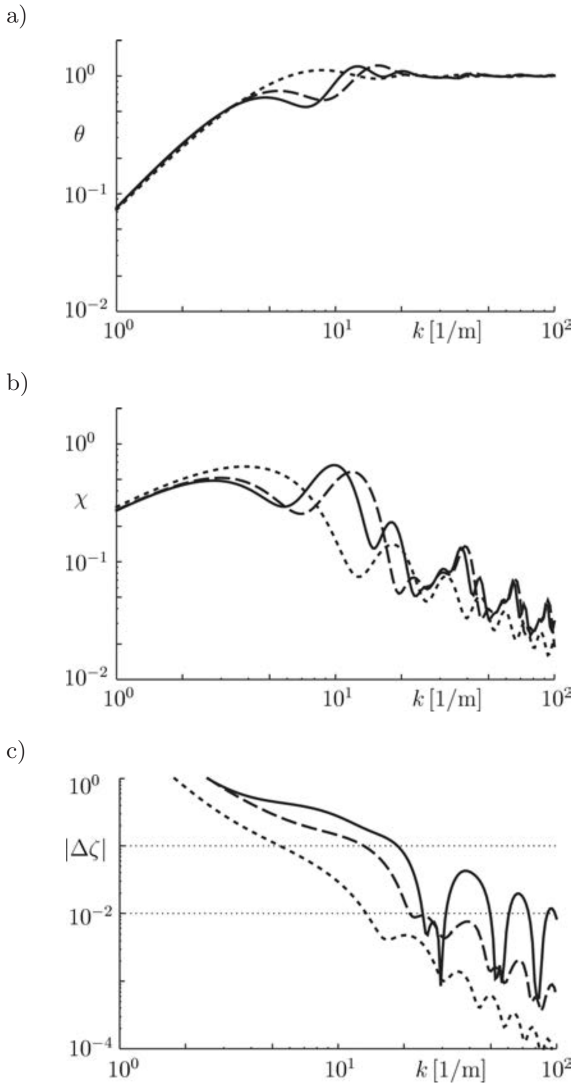

Fig. 5. The normalized acoustic impedance ζ as a function of the acoustic wavenumber k for $s = 2.0$, $S = 0.5\,\mathrm{m}^2$: a) resistance θ, b) reactance χ, c) error $|\Delta\zeta|$. Key: solid line $s_1 = 0.0$, dashed line $s_1 = 0.5$, dotted line $s_1 = 1.0$.

sidering it becomes useless. In the opposite case when $s \to \infty$ and $a_1 \to 0$, we have also that $a \to 0$ which implies that the radius of a cylindrical baffle and its scattering surface both vanish. Investigating the influence of such a vanishing baffle on the radiated acoustic waves is also useless and in this case the formulations valid for a vibrating annular piston located in a flat rigid baffle with no cylindrical baffle should be used instead (cf. Subsec. 2.3. and Merriweather (1969); Stepanishen (1974); Thompson (1971)).

The arrangement of curves on Fig. 5 is similar to the previous figure. Now, the following values of $s = 2$ and $S = 0.5\,\mathrm{m}^2$ have been fixed and the curves as functions of k have been presented for the three different values of s_1. The curves illustrate the acoustic resistance θ (Fig. 5a), the acoustic reactance χ (Fig. 5b) and the absolute error of the acoustic impedance calculated using the asymptotic formulations (Fig. 5c). The analysis of the curves allows determining the influence of the radius of cylindrical circular baffle on the acoustic impedance. The curves prepared for $s_1 = 0.0$ and $s_1 = 0.5$ are the closest to one another according to their shapes. The main difference regards the shift of oscillations as a function of k. This means that the cylindrical baffle influences the acoustic impedance weakly within the interval $0.0 \le s_1 \le 0.5$ for $s = 2.0$. This situation changes considerably when $s_1 \to 1$. The resistance curve is almost free of oscillations as a function of k whereas the reactance curve has its oscillations of much smaller amplitudes than for the remaining values of s_1. The curves are more similar to the corresponding resistance and reactance curves of a vibrating cylindrical circular finite piston located on the infinite cylindrical baffle (Greenspon, Sherman, 1964; Robey, 1955; Thompson, 1967) than to the curves of a vibrating annular piston located on the flat rigid baffle (cf. Merriweather (1969); Stepanishen (1974); Thompson (1971) and the curve for $s_1 = 0.0$). This means that within the interval $0.5 < s_1 \leq 1.0$ for $s = 2.0$ the influence of the cylindrical baffle on the acoustic impedance is essential. Obviously, the biggest influence appears for $s_1 = 1.0$ and this case is the most important when the scattering of acoustic waves on the cylindrical baffle is desirable. The additional feature of the presented asymptotic formulations is that their error decreases relatively fast for $s_1 = 1.0$ which is easy to notice in Fig. 5c. In this case, the error falls below 10^{-2} for k equal to about 15 whereas for $s_1 = 0.0$ (no cylindrical baffle) this occurs for k equal to about 100.

4. Concluding remarks

The results obtained in this study allow claiming that the influence of the cylindrical circular baffle on the scattering of acoustic waves radiated by a vibrating concentric annular piston is essential. The main effect is that the behavior of acoustic impedance of such sound source is similar to the behavior for a vibrating cylindrical circular finite piston located on the infinite cylindrical baffle which occurs for the big values of the radius of the semi-infinite cylindrical baffle when compared with the internal radius of a vibrating annular piston. This means that both kinds of sources are equivalent to one another within the acceptable error bounds under the assumptions made in this study.

Some of the integrals necessary for numerical calculations of the acoustic impedance have been substituted by the exact non integral formulations valid for a vibrating circular piston located in the flat rigid baffle proposed earlier by Stepanishen (1974), Thompson (1971). The remaining integrals have been presented as the asymptotic formulations valid for big values of the acoustic wavenumber.

Appendix A. Analysis of the contour integral

The acoustic impedance has been presented in the integral form in Eqs. (17). In the case of integrating over the contour L_1 it can be formulated as

$$\zeta = \lim_{\substack{R\to\infty\\ \epsilon\to 0}} \left(\int\limits_{\overrightarrow{AB}} + \int\limits_{\overrightarrow{CD}} \right) = - \lim_{\substack{R\to\infty\\ \epsilon\to 0}} \left(\int\limits_{\overset{\frown}{BC}} + \int\limits_{\overset{\frown}{DE}} + \int\limits_{\overrightarrow{EA}} \right), \tag{44}$$

where the corresponding integrand is analytic on the contour and inside it. The Cauchy's theorem and the Jordan lemma have been used to formulate the following limits

$$\begin{aligned}
&\lim_{\epsilon\to 0} \int\limits_{\overset{\frown}{BC}} = \pi \mathrm{i} \operatorname*{Res}_{x=1} \frac{2F(s, s_1, ka_1\sqrt{1-x^2})}{(s^2-1)(1-x^2)} = -1,\\
&\lim_{\substack{R\to\infty\\ \epsilon\to 0}} \int\limits_{\overrightarrow{EA}} = \frac{2\mathrm{i}}{s^2-1} \int\limits_0^\infty F(s, s_1, ka_1\sqrt{1+y^2}) \frac{\mathrm{d}y}{1+y^2},\\
&\lim_{R\to\infty} \int\limits_{\overset{\frown}{DE}} = 0.
\end{aligned} \tag{45}$$

The following expansion terms $J_1(u) \approx u/2$ and $Y_1(u) \approx -2/\pi u$ for $u \to 0$ have been used while calculating the residue in Eq. $(45)_1$ and the substitution $x = -\mathrm{i}y$ has been applied while integrating in Eq. $(45)_2$. Inserting Eqs. (45) into Eq. (44) results in the acoustic impedance from Eqs. (19) in the form of integrals calculated over variable y within the interval from 0 to ∞.

In the case of integrating over the contour L_2 in Eq. (17), it has been obtained that

$$\begin{aligned}
\zeta &= \lim_{\substack{R\to\infty\\ \epsilon\to 0}} \left(\int\limits_{\overrightarrow{AB}} + \int\limits_{\overrightarrow{CD}} \right)\\
&= - \lim_{\substack{R\to\infty\\ \epsilon\to 0}} \left(\int\limits_{\overset{\frown}{BC}} + \int\limits_{\overset{\frown}{DE}} + \int\limits_{\overrightarrow{EF}} + \int\limits_{\overset{\frown}{FA}} \right).
\end{aligned} \tag{46}$$

The individual limits have been formulated as follows

$$\begin{aligned}
&\lim_{\epsilon\to 0} \int\limits_{\overset{\frown}{BC}} = \frac{\pi \mathrm{i}}{2} \operatorname*{Res}_{x=0} \left[\frac{-2}{s^2-1} F(s, s_1, ka_1 x) \frac{1}{x\sqrt{1-x^2}} \right] = -1,\\
&\lim_{\substack{R\to\infty\\ \epsilon\to 0}} \int\limits_{\overrightarrow{EF}} = \frac{2\mathrm{i}}{s^2-1} \int\limits_1^\infty F(s, s_1, ka_1 x) \frac{\mathrm{d}x}{x\sqrt{1-x^2}},\\
&\lim_{R\to\infty} \int\limits_{\overset{\frown}{DE}} = \lim_{\epsilon\to 0} \int\limits_{\overset{\frown}{FA}} = 0.
\end{aligned} \tag{47}$$

Inserting Eqs. (47) to Eq. (46) results in the acoustic impedance in Eq. (19) in the form of integrals calculated over variable x within the interval from 1 to ∞.

Appendix B. Asymptotic formulations using the stationary phase method

The following asymptotic formulation valid for $1 \ll u$ has been used (Watson, 1944)

$$H_1^{(1)}(u) = -(1+\mathrm{i}) \left(1 + \frac{3\mathrm{i}}{8u} \right) \frac{\exp(\mathrm{i}u)}{\sqrt{\pi u}} + \mathcal{O}(u^{-5/2}) \tag{48}$$

for obtaining the asymptotic formulations of integrals in Eqs. (19). Further, the zero asymptotic expansion term has been used in the stationary phase giving (Fedoryuk, 1987)

$$\begin{aligned} I &= \int\limits_{w_0}^{w_0+\epsilon} f(w)\exp[\mathrm{i}bS(w)]dw \\ &= \frac{1}{2}\sqrt{\frac{2\pi}{b\,|S''(w_0)|}}\,[f(w_0)+\mathcal{O}(1/b)] \\ &\quad \cdot \exp\{\mathrm{i}[bS(w_0)+(\pi/4)\ \mathrm{sign}\, S''(w_0)]\}, \end{aligned} \tag{49}$$

where the following values have been assumed $b = 2(1-s_1)ka_1$, $2(s-s_1)ka_1$, $(s-1)ka_1$, $(s+1-2s_1)ka_1$, $S(w) = \sqrt{1+w^2}$, $f(w) = (1+w^2)^{-2}$ and $w_0 = 0$.

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