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Modelling the factors of mine production planning considering the risk free valuation and new cut-off grades algorithm

Introduction

Production planning and choosing the cut-off grades have been the two concerns in mine engineering and lots of efforts have been taken to determine them. The exact determination of the optimum cut-off grades not only specify the destination of the minerals but also affect mines' planning and designing (Dowd and Onur 1992; Hustrulid and Kuchta 1996; Ramazan 2007; Dagdelen 1993; Bascetin and Nieto 2007; Dagdelen and Kazuhiro 2007; Boland et al. 2009; Asad and Topal 2011; Asad and Dimitrakopoulos 2013; Sattarvand and Niemann-Delius 2013; Moosavi et al. 2014). Regarding to the economic and technical importance of optimum cut-off grades, different algorithms and various numerical and analytical optimizing methods (Rahimi et al. 2014) have been applied. In these optimizing methods, different purposes have come into consideration so as to figure out the optimum amounts of these cut-off grades. Furthermore, IRR and the Output Rate have been regarded as the target function (Khodayari et al. 2012; Rendu 2009; Rahimi et al. 2015). Lane (1964, 1988) first researched about the determination of the optimum cut-off grades. The NPV and the profit optimization were regarded as his intended aim functions. Although Lane's results have been remarked as the first fundamental findings in this regard, they actually included some failures. For instance, he ignored the effect of the following items on determining the optimum cut-off grades.

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The main failures of this method include: the effect of different processing methods on the amount of mines' cut-off grades, the influence of fluctuations and changes of the final product's selling price on the economic index of the mines, the investment and operational costs of different processing methods, the environmental consideration of mines and reconstructing them, ignoring the increase and decrease of the price and considering the recovery and the effect of environmental pollutants of the processing methods.

Some attempts have been taken to solve the failures caused by Lane's algorithm. Therefore, in addition, environmental considerations the effect of reclamation costs on the amount of cut-off grades has been evaluated (Rashidinejad et al. 2008). On the other hand, Asad (2007) tried to introduce the price increase in the algorithm of the optimum cut-off grades to scrutinize the influence of the price fluctuations on the algorithm determining the cut-off grades. Bastian also added the cost of lost opportunity as an optimization factor into the algorithm of the cut-off grades. In some cases, other methods and optimizing algorithms were practiced to determine the optimum cut-off grades each applied to precisely specify the cut-off grade including the determination of the grades of multi-metal stockpiles (Osanloo and Ataee 2003), the grades of underground mines and the determination of underground mines (Johnson et al. 2011). However, the studies didn't consider the effect of different processing methods and price prediction on the amounts of optimum cut-off grades. Moreover, the process recovery of the processing methods has been remarked constantly in all the research done in the calculation of the cut-off grades. On the other hand, the effect of the grade changes on the recovery amount of the pyro-metallurgy and hydrometallurgy methods has been ignored. The role of the price predicting model is more prominent when the market of the minerals and their products is exposed to high fluctuations. Hence, it is not possible to develop a certain principle to determine the optimum cut-off grades in long term and plan the production.

The technical and economic modeling of mines gets complete unless the selling price of the final product is included. Lots of research has been done to predict the price of the final product using different methods and modeling. However, the roles of this modeling and price prediction have been ignored in the calculations related to the optimum cut-off grades. In fact, there is no comprehensive algorithm evaluating the effect of the fluctuations and the price prediction modeling on determining the optimum cut-off grades and calculating the NPV. Regarding the assessment of the risk indices of price fluctuations, the amount of UPL and production planning has been calculated using dynamic methods. Also, several comprehensive algorithms have been presented which are able to directly include the effect of the price prediction on the amount of the economic indices.

According to the effect of the selling price of the final product on the economic indices of mines and the optimum cut-off grades, it was essential to introduce its related parameters into calculations associated with the optimum cut-off grades. For this purpose, one model was done to evaluate the maximum NPV and the optimum cut-off grades considering various predicting models for a mine. This algorithm is unique and prominent in that it can upgrade the calculation related to the cut-off grades by entering different price prediction modeling.

1. Material destination and price fluctuation

The cut-off grade is defined as an index figuring out the destination of the minerals. From the technical point of view, it is concluded that the sending of minerals is a function of the processing method and the types of its minerals. From the economic view point, it is obvious that the most important economic indices are concerned with determining the cut-off grades regarding the changes of metals price and the selling income. As a result, they should be an effect on the calculations. It is significant to have a suitable economic and technical concept and pattern due to the type of ore determining the cut-off grades. Regarding the technical aspect, it is assumed that all types of the copper ore can be processed by treatment methods. The first type of copper ore describes a mine containing only oxidation zones. The second type is about supergene and hypogene zones. And third one contains both oxide and sulfide copper ores in transition zones. Oxide and sulfide deposits do not depend on each other and their processing methods are different, the cut-off grades of these deposits can be calculated individually. Chalcopyrite (CuFeS_2) and Bornite (Cu_5FeS_4) are recognized as the primary sulfide minerals which can be generally processed by Pyrometallurgical methods. Chalcocite (Cu_2S) and Covellite (CuS) are considered as secondary sulfide minerals which are easily leached with sulfuric acid if an oxidant is present. Secondary copper sulfide minerals can also be processed by pyrometallurgical methods. Malachite ($\text{Cu}_2(\text{CO}_3)(\text{OH})_2$) is an oxidation zone which can be processed by Hydrometallurgical methods such as leaching, solvent extraction and electro winning. Having best practices to set cut-off grade mines for different processing methods is so important.

From the economic perspective, the determination of the cut-off grade relies on various parameters. The most significant parameters specifying and reducing the risk of determining cut-off grades include considering the price and its prediction method and its algorithms. Thus, one algorithm has been presented to dwindle the risk of determining the cut-off grades and destination of the minerals. On the one hand, this algorithm should be able to decrease the risk by assessing the changes and fluctuations of the metals market. On the other hand, it is expected to include the above mentioned technical indices and remark the destination of the minerals in itself.

2. Objective function

For the evaluations of forecasting the metal price to trend method, three equations are listed here to predict copper prices. These three forecasting price models are based on three types of prediction algorithm methods that are as follows: monthly, every 6 months, annually.

Based on these three prediction models, mine planner aimed to begin in the year 2018, has three possible copper scenarios. The first model predicted price, copper price forecast is USD 6,971/ton. The second model predicted price, copper price forecast USD 6,762/ton and the third model predicted price, copper price forecast USD 6,678/ton. So the price of

Table 1. Three types of predication algorithm with time series monthly, six-monthly, yearly

Tabela 1. Trzy typy algorytmów predykcyjnych z szeregami czasowymi miesięcznymi, sześciomiesięcznymi i rocznymi

	Monthly	Every 6 months	Annually
Linear	$y = 0.87x - 29594$ $R^2 = 0.45$	$y = 0.89x - 30278$ $R^2 = 0.46$	$y = 63.2x - 125022$ $R^2 = 0.32$
Polynomial	$y = -9E - 08x^3 + 0.01x^2 - 379.87x + 5E + 06$ $R^2 = 0.75$	$y = 6E - 15x^5 - 1E - 09x^4 + 9E - 05x^3 - 3.3173x^2 + 61993x - 5E + 08$ $R^2 = 0.80$	$y = -0.12x^5 + 1246.4x^4 - 5E + 06x^3 + 1E + 10x^2 - 1E + 13x + 4E + 15$ $R^2 = 0.71$
Logarithmic	$y = 35355\ln(x) - 369180$ $R^2 = 0.46$	$y = 35868\ln(x) - 374659$ $R^2 = 0.48$	$y = 126881\ln(x) - 963027$ $R^2 = 0.32$

y – price forecast for copper (Dollars upon metric tons).

x – the period of time elapsed from the first data considered in the analysis (from the year 2000 to 2025).

copper on the above date could be between USD 6,678 to 6,971/ton. In practice, the mine planner with several projects is based on a broad range of possible price scenarios. So, the modeling of optimum cut-off grades determination can be developed. Different mineral processing methods and price prediction model are considered in the presented model. In the event the objective function of the mentioned model is to maximize NPV, it is necessary to calculate discounted annual cash flows.

The net present value can be achieved in a discounted series of cash flows as below:

$$NPV = \int_{i=T_o}^{T_{life}} CashFlow_i \cdot (1 + \partial)^{-i} \quad (1)$$

Where, i represents the annuals of the project in the entire mining time and is the discount rate, T_o is the starting time of the project and indicates the project time.

The total marketable product is achieved by applying both leaching and concentration methods from the relation below:

$$\sigma_{Si}(g) = \eta_C \eta_S \cdot \bar{a}_C(g) \cdot \sigma_{Ci}(g) \quad \text{and} \quad \sigma_{Xi}(g) = \eta_H \eta_X \cdot \bar{a}_H(g) \cdot \sigma_{Hi}(g) \quad (2)$$

- g – grade indicators,
- σ_S – pyrometallurgical refined materials (tons),
- σ_X – hydrometallurgical refined materials (tons),
- σ_C – represents the tonnage sent to concentration,
- σ_H – indicates the ore tonnage sent to heap,

- \bar{a}_C and \bar{a}_H – the average grades of the ore sent to concentration and heap, respectively,
 η_C – concentration recovery,
 η_S – smelter and electrorefining processes recovery,
 η_H – indicates heap leaching recovery,
 η_X – the SX and EW processes recovery.

Finally, the NPV in the discounted series of cash flows is calculated as below:

$$\begin{aligned}
 NPV = \max : & \sum_{i=t_0}^{T_{life}} [(f(p_i) - s_i) \eta_C \eta_S \cdot \bar{a}_C(g) \cdot \sigma_{C_i}(g) + \\
 & + (f(p_i)_i - x_{sxi} - x_{ewi}) \eta_H \eta_X \cdot \bar{a}_H(g) \cdot \sigma_{H_{C_i}}(g) - c_i \cdot \sigma_{C_i}(g) - \\
 & - h_i \cdot \sigma_{H_i}(g) - e_i \cdot \sigma_{E_i}(g) - (f_i^c + f_i^h + \partial \cdot NPV_i)] \cdot (1 + \partial)^{-i}
 \end{aligned} \quad (3)$$

- $f(p_i)$ – indicates the commodity price function as based on the perdition model,
 s – the smelting production costs,
 x_{sx} and x_{ew} – indicate the marketing, SX and EW annual processing costs,
 c – introduces the ore concentration cost,
 h – the ore leaching operating cost, is the mining cost,
 σ_E – represents all of the ore extracted from the mine,
 f^c – indicates the fixed cost of concentration and smelting processes and
 f^h – the fixed cost of heap leaching, SX and EW processes.

Environmental parameters play key roles in cut-off grades determination.

The environmental problems of hydrometallurgical methods are also indicated in Table 2.

Table 2. The environmental problems of heap leaching methods and associated costs

Tabela 2. Problemy środowiskowe metod ługowania hałdy i związanych z nimi kosztów modelu predykcyjnego (i = rok i j = etap iteracji)

Potential of pollution	The environmental problems of heap leaching methods		
	leached waste remained on heap	SX and EW tailings	hydrometallurgical process wastes
Environmental effects	Acid drainage from leached wastes, heap sliding, water contamination by wastes, ...	Dumping of SX and EW tailings, chemical pollutions if tailings, residues and waste solution, water contamination by tailing, ...	Air pollutions by acid vapor, non-recyclable materials production, sewerage pollution, beed water production, anode scraps and ...

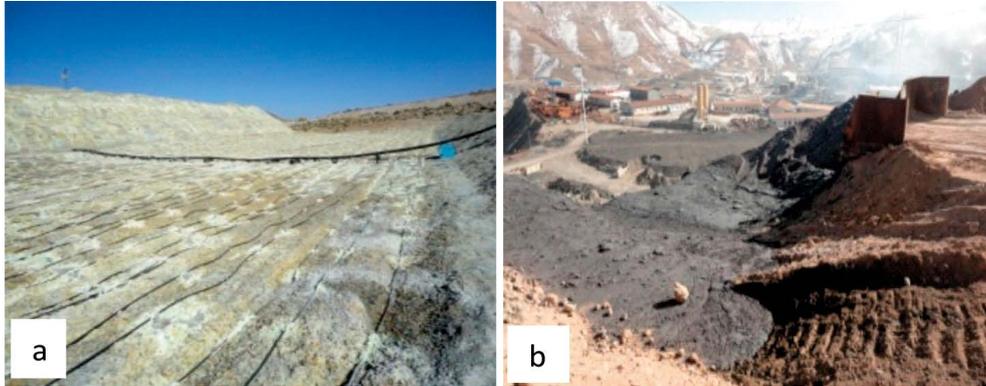


Fig. 1. a) The pad leaching irrigated by sulfuric acid,
 b) The copper slag dumped in the valley and environmental effects

Rys. 1. a) Plac ługowania kwasem siarkowym,
 b) Żużel miedziowy składowany w dolinie i wpływ na środowisko

Since the objective function is the constrained optimization problem; the Lagrange multiplier method can be applied to solve the cut-off grades optimization function. Hence, five capacities are considered as limiting factors and the Lagrangian function of net present value can be written as follows:

$$\begin{aligned}
 \mathcal{L}(g^h, g^c, \lambda) = & \sum_{g=g^c}^G (f(p_i)_i - s_i) \eta_C \eta_S \cdot g \cdot q_i(g) + \\
 & + \sum_{g=g^h}^{g^c} (f(p_i)_i - x_{sxi} - x_{ewi}) \eta_H \eta_X \cdot g \cdot q_i(g) - \sum_{g=g^c}^G c_i \cdot q_i(g) - \\
 & - \sum_{g=g^h}^{g^c} h_i \cdot q_i(g) - \sum_0^G e_i \cdot q_i(g) - (f_i^c + f_i^h + \partial \cdot NPV_i) \cdot T + \sum_{k=1}^m \lambda_k T_k(g)
 \end{aligned} \tag{4}$$

- ↪ g^h, g^c – indicated the cut-off grades of the leaching and concentration methods.
- q – represents the tonnage extracted from the mine,
- λ – the Lagrange multiplier,
- \mathcal{L} – the Lagrangian indicator,
- k – the constraint number indicator,
- m – the number of constraint,
- G – the maximum ore grade,
- T – project time; is also controlled by mine and plants capacities.

The lagrangian equations set is solved. Since the plants and mining capacities are individually considered as a limiting factor, concentration and heap leaching cut-off grades can be determined.

Considering the mine output rate as the limiting factor, the optimum cut-off grade is defined as follows:

$$\text{mining limitation: } \left\{ \begin{array}{l} g_E^h = \frac{h_i}{\eta_H \eta_X (f(p_i) - x_{sxi} - x_{ewi})} \\ g_E^c = \frac{c_i - h_i}{(f(p_i)_i - s_i) \eta_C \eta_S - \eta_H \eta_X (f(p_i)_i - x_{sxi} - x_{ewi})} \end{array} \right. \quad (5)$$

The result of this process, the risk due the price uncertainty can be modeled by the effects of price volatility on the cut-off grade and deposit.

3. Optimization algorithm

The optimization algorithm determining the amount of maximum NPV and cut-off grades is explained below regarding price prediction algorithms. As observed in the following flowchart, the primary amount of the exploration stockpile is added into mode-

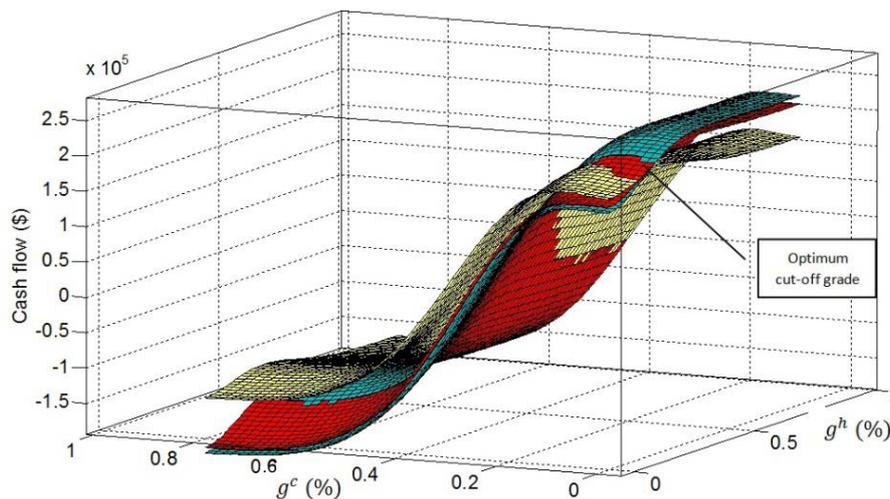


Fig. 2. The 3-D schematic figure for determining the optimize cut-off grades

Rys. 2. Trójwymiarowa schematyczna ilustracja do określania optymalnych wartości brzeżnych

ling and then, the productive capacities of different plants are specified. For the next step, the price prediction function is included in the algorithm. The amount of the cut-off grades, as a primary prediction, is calculated by mathematical modeling as observed in the flowchart. Consequently, determining the amount of the primary NPV is noted in the algorithm.

The figure below illustrates the 3D approach to determine the initial optimum cut-off grades as based on the maximum cash flow.

Then, the optimization of NPV is assessed. If it is not maximum, the algorithm calculates new cut-off grades with a new NPV. If the amount of the NPV is maximum, the mining and calculations are postponed to the following year. This process is kept on to access the NPV and the amount of optimum grade.

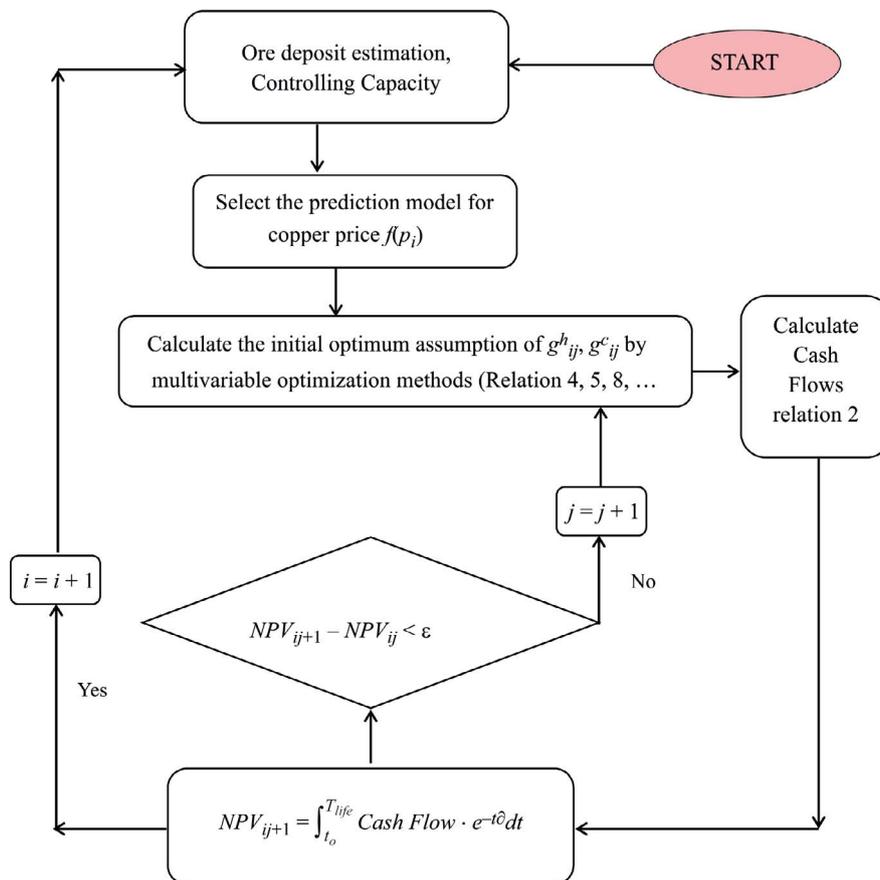


Fig. 3. The iteration flowchart containing the initial amount of the cut-off grades and prediction model (i = year and j = Iteration step)

Rys. 3. Schemat blokowy iteracji zawierający początkową ilość wartości granicznych

4. Results and discussion

The hypothetical mine is contained in the primary and secondary copper sulfide ores. The grade-tonnage distribution of the deposit is shown in Table 3. It is considered that 49.5 million tons of material is mined in 3 push backs.

Table 3. The grade distribution of the hypothetical mine

Tabela 3. Rozkład ocen hipotetycznej kopalni

Grade (%)	Push back 1 (ton)	Push back 2 (ton)	Push back 3 (ton)
0–0.1	1,854,300	2,725,480	4,210,020
0.1–0.2	4,948,725	6,415,040	6,706,235
0.2–0.3	3,848,700	4,959,804	5,331,496
0.3–0.4	4,150,540	2,005,040	209,720
0.4–0.5	1,212,650	503,580	85,170
0.5–0.6	110,600	204,875	16,045
0.6–0.7	16,400	5,840	18,089
G > 0.7	2,202	589	240
Sum	16,144,117	16,820,248	16,577,015

It is necessary to perform a price prediction algorithm in the NPV optimization model and cut-off grades in order to perform the optimization algorithm. Thus, three price prediction models have been presented below including: linear, polynomial and exponential. The model has predicted the prices from 2000 to 2025 based on the price prediction model. As observed in Figures 4 to 6, the chronological periods related to prediction are considered monthly, every 6 months and annually. Hence, it is expected to precisely predict the price of copper according to the consecutive changes of the copper price within a year. As the price of copper has not been changed for years, it is expected that the prediction is done more precisely. The results of the prediction model have been added to the optimization algorithm of flow chart 3 and are made to calculate the NPV and the cut-off grades. From the year 2000 to 2018, the price information has been based on the information from the CRU site.

From the three models presented in figures including linear, exponential and polynomial, the polynomial model scored the highest R^2 . Thus, this prediction model has been used in the case study example and also used in the optimization algorithm. It should be pointed out that using other prediction models might be directed into more NPV, but they are less precise. In real models, it is better to use the most compatible model.

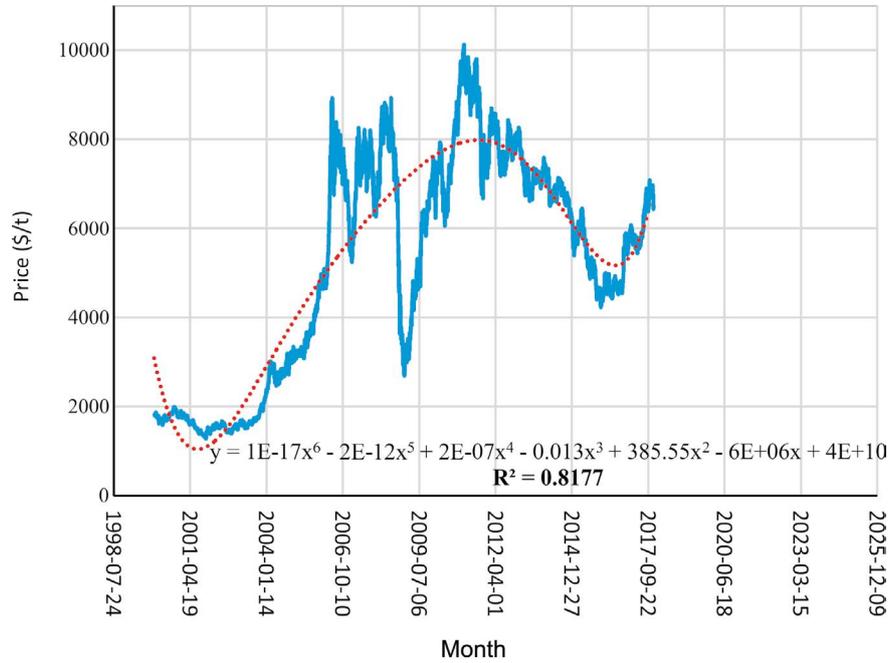


Fig. 4. Regression obtained from the monthly price of copper

Rys. 4. Regresja uzyskana z miesięcznej ceny miedzi

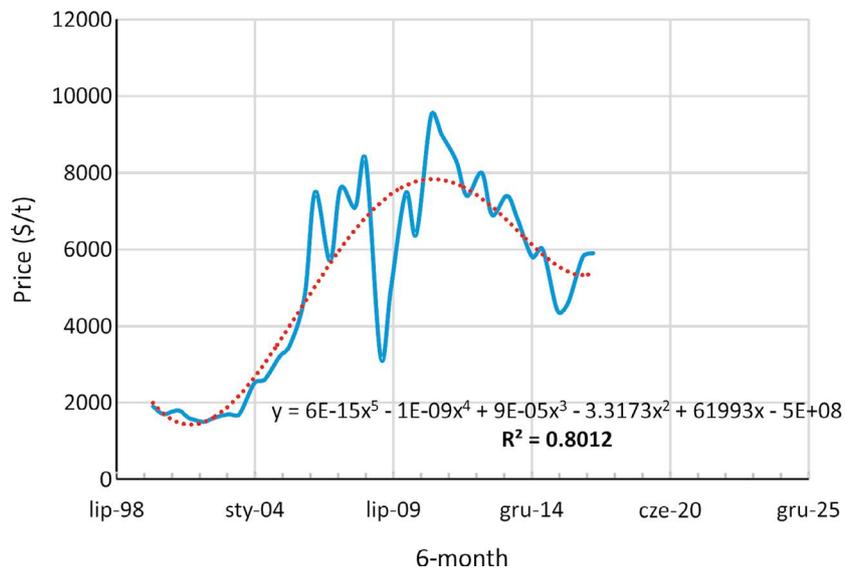


Fig. 5. Regression obtained from the six-month price of copper

Rys. 5. Regresja uzyskana z sześciomiesięcznej ceny miedzi

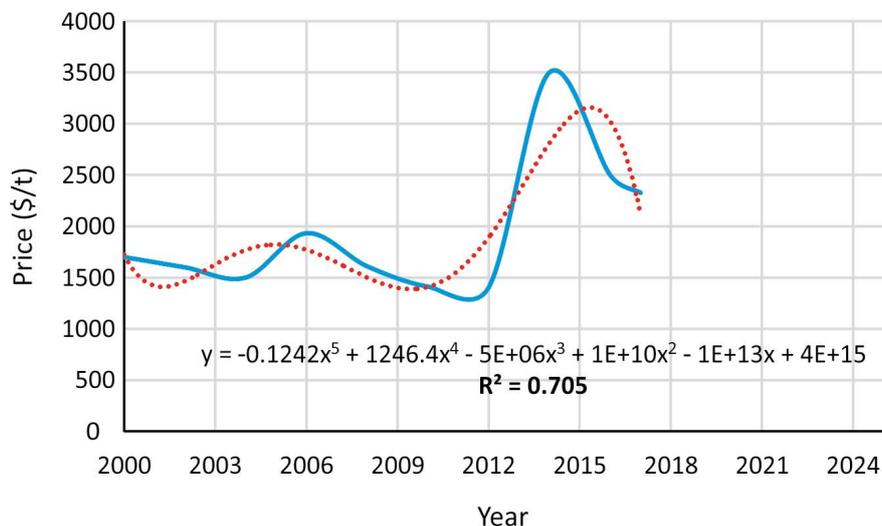


Fig. 6. Regression obtained from the annual price of copper

Rys. 6. Regresja uzyskana z rocznej ceny miedzi

The assumed optimum cut-off grade comes into the iterative algorithm and final optimum cut-off grades are calculated. An iterative process must be employed since an unknown NPV is present in the NPV maximization problem. The unknown NPV depends upon the opportunity cost and the opportunity cost is inevitable when constraints in the cash flow model are present. This cost occurs when the materials were not previously scheduled to be mined or processed. The presented algorithm and computer programming are able to determine limiting, dual and triple balancing cut-off grades and develop an optimum cut-off grades policy in several iterations. The final results of the iteration algorithm and cut-off grades optimization process are outlined in Table 3. As shown in Table 4, the cut-off grades, and maximum NPV are presented.

As indicated in Table 4, the amount of NPV and other economic indices of mines can be calculated using the prediction algorithm and modeling the optimum cut-off grades. The following figure illustrates the relationships of these variables as graphs due to the effect of metal price changes on the amount of NPV and cut-off grades.

As observed in Figure 4, changes in the price of copper from USD 4300 to 8300 ton occur from the 2000 to 2025. This is directed into changing the optimum cut-off grades from 0.06% to 0.47%. In addition, this change affects the amount of cash flows and the results presented in Table 3. Fig 4 analyzes the amount of NPV due to its direct and indirect effect on the price fluctuations. As observed, the amount of maximum NPV has been increased from USD 36 M to 127 M through changing the cut-off grades of all the processing methods. Due to the price prediction model, it is expected that the amount of the optimum cut-off grades of this hypothetical mine is predicted by polynomial methods and its NPV is predicted as well.

Table 4. The results of execution optimization model

Tabela 4. Wyniki modelu optymalizacji

Year	Pushback	Total material (TT)	σ_{Ei} (T_m)	σ_{Si} (T_p)	σ_{Xi} (T_p)	g^c (%)	g^h (%)	Life (year)	Cash Flows (\$)	Overall NPV (\$)
1	1	49,541,380	4,048,234	4,775	459	0.21	0.18	12.2	16,654,445	84,223,100
2	1	45,493,146	4,034,201	4,769	458	0.21	0.18	11.3	16,432,442	79,436,674
3	1	41,458,945	4,019,380	4,763	457	0.21	0.17	10.3	15,443,565	73,647,876
4	1	37,439,565	4,003,824	4,756	455	0.21	0.17	9.4	15,008,321	68,423,435
5	1	33,435,742	38,479	46	4	0.20	0.17	0.1	37,204	62,455,009
5	2	33,397,263	3,949,173	4,704	449	0.20	0.17	8.3	14,818,331	56,433,654
6	2	29,448,090	3,970,421	4,742	451	0.20	0.17	7.4	14,342,134	50,432,054
7	2	25,477,668	3,952,462	4,735	450	0.20	0.17	6.4	14,000,143	43,321,543
8	2	21,525,206	3,933,665	4,727	448	0.20	0.17	5.5	13,324,543	38,546,678
9	2	17,591,540	1,014,525	1,223	115	0.20	0.17	1.2	947,988	30,543,600
9	3	16,577,015	2,899,526	3,495	330	0.20	0.17	3.3	11,709,361	23,567,432
10	3	13,677,489	3,893,379	4,709	443	0.20	0.17	3.5	12,113,872	16,543,986
11	3	9,784,109	3,872,047	4,700	441	0.20	0.17	2.5	11,761,543	10,342,431
12	3	5,912,062	3,849,663	4,690	438	0.20	0.17	1.5	10,433,097	4,455,767
13	3	2,062,399	2,062,399	2,525	235	0.20	0.17	0.5	8,432,265	112,211

TT – thousand tons, T_p – tons of product, T_m – tons of material.

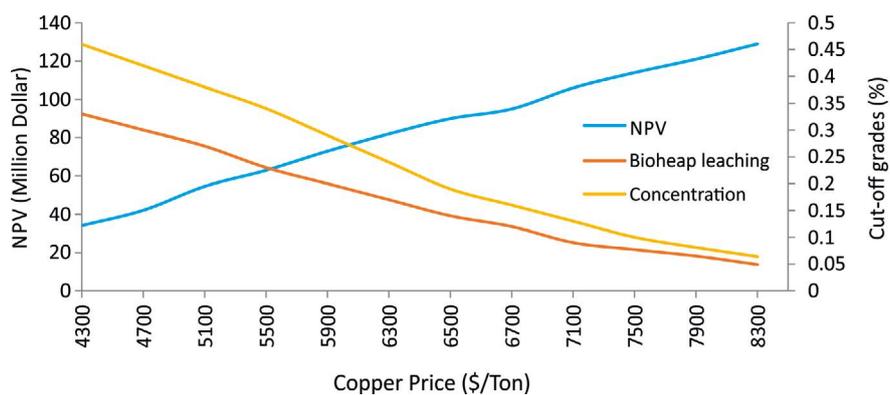


Fig. 7. The variation of NPV and cut-off grades versus copper price changes

Rys. 7. Zmienność NPV i wartości granicznych w funkcji zmian cen miedzi

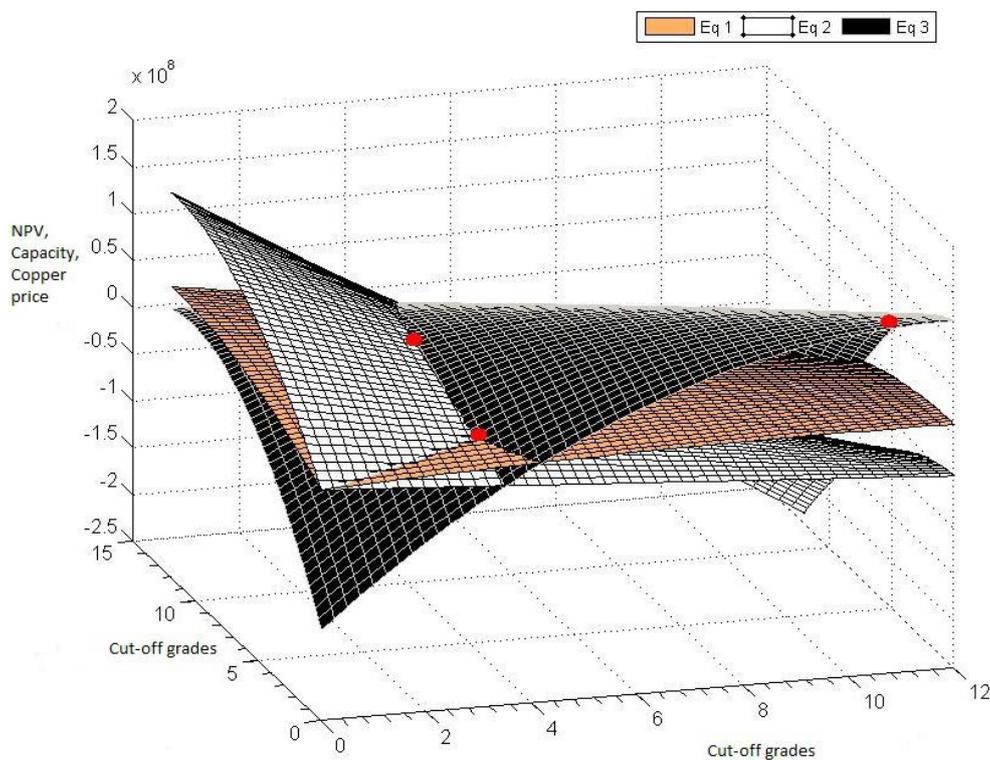


Fig. 8. The cut-off grades versus NPV and copper price (Black surface: prediction model 1), (Brown surface: prediction model 2) (White surface: prediction model 3)

Rys. 8. Wartości graniczne w funkcji NPV i ceny miedzi (czarna powierzchnia: model predykcji 1), (powierzchnia brązowa: model predykcji 2) (powierzchnia biała: model predykcji 3)

The effect of the selling price of copper cathode on the economic indices of hypothetical mine is investigated. As expected, the cathode price has contributed to change the economic indices for the sake of high benefit. Conclusively, the following graphs figure out the influence of changing the selling price on cut-off grade, NPV of the plant.

As observed, the fluctuation of selling price of prediction model (Eq1 to 3) remarkably affects the cut-off grades. The increase of these prices the cut-off grades of different methods are changed in the mine. This figure reveals that the fluctuation of selling price highly influences the change of cut-off grades.

Conclusions

Many researchers and miners have been scrutinizing the economic indices of mines for consecutive years. Several methods have been evaluated for this purpose. Most of these

methods deal with calculating the economic indices such as cut-off grades and UPL in order to maximize the benefit. The algorithms presented in other studies are not able to assess the price changes and income for the calculation of these indices. In the analysis of the hypothetical mine, the copper price prediction algorithm has been affected on the calculations related to determining the optimum cut-off grades and the NPV. Moreover, the maximum amount of NPV has been evaluated through considering these changes and price fluctuations. This algorithm includes the iteration algorithm presented in Figure 3. The primary results demonstrate that the price changes and its prediction algorithm affected the amount of the optimum cut-off grades. Therefore, the effect of the prediction algorithm on the amount of the optimum cut-off grades has decreased these grades in the last years of this mine. This is while the introduction of the price prediction algorithm reduces the amount of cut-off grades from 0.06% to 0.47%. This decrease is mostly caused by the effect of the simultaneous decrease of the cost of the lost opportunity and also the increase of the price caused by the prediction of the product selling. It is concluded that the cash flows have been increased up to USD 16M by the income increase made by the prediction of the final product selling. It also changed the calculation method of the NPV. The changes of the copper price and its prediction made some changes in the optimum amount of the productive economic cathode. In fact, no specific trend has been observed in the production amount for several years. These fluctuations are caused by the economic interval of the final product by price changes. The assessment of the price fluctuations and its changes are directed into precisely calculate the algorithms. Hence, considering the predicted algorithms contributes to accessing the real amounts and economically analyzing the mines.

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**MODELLING THE FACTORS OF MINE PRODUCTION PLANNING CONSIDERING
THE RISK FREE VALUATION AND NEW CUT-OFF GRADES ALGORITHM**

Abstract

The average grades of copper mines are dropped by extracting high grade copper ores. Based on the conducted studies in the mine field, the uncertainty of economic calculations and the insufficiency of initial information is observed. This matter has drawn considerations to processing methods which not only extracts low grade copper ores but also decreases adverse environmental impacts. In this research, an optimum cut-off grades model is developed with the objective function of Net Present Value (NPV) maximization. The costs of the processing methods are also involved in the model.

In consequence, an optimization algorithm was presented to calculate and evaluate both the maximum NPV and the optimum cut-off grades. Since the selling price of the final product has always been considered as one of the major risks in the economic calculations and designing of the mines, it was

included in the modeling of the price prediction algorithm. The results of the algorithm performance demonstrated that the cost of the lost opportunity and the prediction of the selling price are regarded as two main factors directed into diminishing most of the cut-off grades in the last years of the mines' production.

Key words: NPV maximization, cut-off grade, selling price, price prediction

**MODELOWANIE CZYNNIKÓW PLANOWANIA PRODUKCJI GÓRNICZEJ Z UWZGLĘDNIENIEM
WYCENY BEZ RYZYKA I NOWEGO ALGORYTMU WARTOŚCI BRZEŻNYCH**

Streszczenie

Rudy Cu średniej jakości są tracone poprzez wydobywanie rud miedzi o wysokiej jakości. Na podstawie przeprowadzonych badań w obszarze kopalni zaobserwowano, że niepewność obliczeń ekonomicznych jest powodem niewystarczalności informacji początkowych. W tej sprawie zwrócono uwagę na metody przetwarzania, które nie tylko wydobywają rudy miedzi o niskiej jakości, lecz także zmniejszają niekorzystny wpływ na środowisko. W artykule opracowano optymalne modelowanie wartości brzeżnych z obiektywną funkcją maksymalizacji wartości bieżącej netto (NPV). Koszty metod przeróbki są również uwzględnione w modelu. W związku z tym przedstawiono algorytm optymalizacji w celu obliczenia i oceny zarówno maksymalnego NPV, jak i wartości granicznych. Ponieważ cena sprzedaży produktu końcowego zawsze była uważana za główne ryzyko w obliczeniach ekonomicznych i projektowaniu kopalń, dlatego też została uwzględniona w modelowaniu algorytmu przewidywania cen. Wyniki działania algorytmu pokazały, że koszty utraconej szansy i przewidywanie ceny sprzedaży są uważane za dwa główne czynniki zmierzające w większości do zmniejszenia wartości granicznych produkcji kopalń.

Słowa kluczowe: maksymalizacja NPV, wartości brzeżne, cena sprzedaży, prognoza cen