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A 4-D chaotic hyperjerk system with a hidden attractor, adaptive backstepping control and circuit design

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A novel 4-D chaotic hyperjerk system with four quadratic nonlinearities is presented in this work. It is interesting that the hyperjerk system has no equilibrium. A chaotic attractor is said to be a *hidden attractor* when its basin of attraction has no intersection with small neighborhoods of equilibrium points of the system. Thus, our new non-equilibrium hyperjerk system possesses a hidden attractor. Chaos in the system has been observed in phase portraits and verified by positive Lyapunov exponents. Adaptive backstepping controller is designed for the global chaos control of the non-equilibrium hyperjerk system with a hidden attractor. An electronic circuit for realizing the non-equilibrium hyperjerk system is also introduced, which validates the theoretical chaotic model of the hyperjerk system with a hidden chaotic attractor.

Key words: chaos, chaotic systems, hyperjerk systems, hidden attractors, adaptive control, backstepping control, circuit design

1. Introduction

Various dynamical systems with chaos have been reported in many modelling applications [1–4]. A number of authors have considered applications of chaos in different fields [5–16]. In [5–7], Vaidyanathan discussed chaos control and synchronization of novel 3-D chemical chaotic reactors. In [8–11], Vaidyanathan discussed Cellular Neural Network (CNN) attractors, FitzHugh-Nagumo neurology models, Tokamak systems and Lotka-Volterra population biology models.

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In [12], Vaidyanathan *et al.* discussed backstepping controller design for the anti-synchronization of WINDMI chaotic systems. In [13], Cavusoglu *et al.* proposed a new chaotic system with high dynamic features and designed chaos-based hybrid RSA (CRSA) encryption encryption algorithm design in which RNG and RSA algorithms were used together. In [14], Ravi *et al.* proposed two 3-stage hybrid prediction models to address financial time-series prediction problem. In [15], Khorashadizadeh and Majidi proposed a new method for secure communication based on chaos synchronization using Fourier series expansion for compensation of uncertainties. In [16], Hawchar *et al.* proposed an efficient method for time-variant reliability analysis using principal component analysis and polynomial chaos expansion.

A *jerk equation* is given by an explicit third-order differential equation in Classical Mechanics that describes the dynamics of a single scalar variable x (displacement) and having the general form as

$$\frac{d^3x}{dt^3} = f\left(x, \frac{dx}{dt}, \frac{d^2x}{dt^2}\right). \tag{1}$$

The ODE (1) is called a *jerk equation* because the consecutive derivatives of the displacement (x) are velocity $(\dot{x})$, acceleration $(\ddot{x})$, and jerk $(\dddot{x})$ in Classical Mechanics.

A generalization of the jerk dynamics (1) yields the higher-order differential equation

$$\frac{d^{(n)}x}{dt^n} = f\left(x, \frac{dx}{dt}, \ldots, \frac{d^{(n-1)}x}{dt^{n-1}}\right), \quad (n \geqslant 4). \tag{2}$$

The ODE (2) is called a *hyperjerk* equation since it features time derivatives $\frac{d^{(j)}x}{dt^j}$, $(j = 1, \ldots, n-1)$ of a jerk function.

By denoting

$$x_1 = x, \quad x_2 = \frac{dx}{dt}, \quad \ldots, \quad x_n = \frac{dx^{(n-1)}}{dt^{n-1}}, \tag{3}$$

we can also represent the ODE (2) as

$$\begin{cases} \dot{x}_1 &= x_2\,, \\ \dot{x}_2 &= x_3\,, \\ \vdots & \vdots \ \vdots \\ \dot{x}_{n-1} &= x_n\,, \\ \dot{x}_n &= f(x_1, x_2, \ldots, x_n). \end{cases} \tag{4}$$

We call the nonlinear system of differential equations (4) as a *hyperjerk system*, when $n \geqslant 4$.

Several attempts have been made to discover chaotic hyperjerk system [17]. Elementary chaotic hyperjerk flows were found by Munmuangsaen and Srisuchinwong [18]. By applying the concept of memory element, Bao *et al.* presented a simple chaotic memory system with complex dynamics [19]. In [20], Dalkiran and Sprott constructed a hyperjerk system with exponential nonlinear function. Four-dimensional hyperchaotic hyperjerk systems were reported in [21, 22]. All these hyperjerk systems have equilibrium points and the stability types of the unstable equilibrium points of these systems have been discussed as well. We remark that there has been no detailed investigation of hyperjerk systems without equilibrium. In this work, we propose a new non-equilibrium hyperjerk system and discuss its properties, circuit design and applications.

A chaotic attractor is said to be a *hidden attractor* when its basin of attraction has no intersection with small neighborhoods of equilibrium points of the system [23]. Thus, our new non-equilibrium hyperjerk system possesses a hidden attractor. Chaotic systems with hidden attractors have applications in many areas [24–28].

Controlling the state trajectories of chaotic systems have received good attention in the control literature [29–33]. We use adaptive backstepping control [34–38] for the global chaos control of the new non-equilibrium hyperjerk system.

2. A non-equilibrium hyperjerk system

In this work, we introduce a novel hyperjerk equation given by the dynamics

$$\frac{\mathrm{d}^4x}{\mathrm{d}t^4} = a\frac{\mathrm{d}^3x}{\mathrm{d}t^3} + bx\frac{\mathrm{d}x}{\mathrm{d}t} + x\frac{\mathrm{d}^3x}{\mathrm{d}t^3} + c\left(x^2 + x\frac{\mathrm{d}^2x}{\mathrm{d}t^2} + 1\right). \tag{5}$$

We define new state variables as

$$\begin{cases} x_1 = x, \\ x_2 = \dot{x}, \\ x_3 = \ddot{x}. \end{cases} \tag{6}$$

Using (6), our novel hyperjerk equation (5) can be rewritten in system form as

$$\begin{cases} \dot{x}_1 = x_2, \\ \dot{x}_2 = x_3, \\ \dot{x}_3 = x_4, \\ \dot{x}_4 = ax_4 + bx_1x_2 + x_1x_4 + c(x_1^2 + x_1x_3 + 1), \end{cases} \tag{7}$$

where a, b and c are positive parameters.

It is interesting to see that the system (7) is chaotic for the following set of parameters: $a = 2.2$, $b = 5$ and $c = 2$.

For $(a,b,c) = (2.2,5,2)$, the Lyapunov exponents of the hyperjerk system (7) are determined by using Wolf's algorithm [43] as $L_1 = 0.1557$, $L_2 = 0$, $L_3 = -1.3455$ and $L_4 = -3.9844$.

The equilibrium points of the hyperjerk system (7) are determined by solving the following system of equations:

$$\begin{cases} x_2 = 0, \\ x_3 = 0, \\ x_4 = 0, \\ ax_4 + bx_1x_2 + x_1x_4 + c(x_1^2 + x_1x_3 + 1) = 0. \end{cases} \tag{8}$$

From the first three equations in Eq. (8), we obtain

$$x_2 = 0, \qquad x_3 = 0, \qquad x_4 = 0. \tag{9}$$

Substituting above in the last equation in Eq. (8), we obtain

$$c(x_1^2 + 1) = 0. \tag{10}$$

Because $c > 0$, it is immediate that the system (7) has no equilibrium point. Thus, we deduce that (7) is a chaotic hyperjerk system with a hidden attractor [23].

It is noted that the initial values of the chaotic non-equilibrium hyperjerk system (7) are taken as $X(0) = (0,-5,0,0)$ for our numerical simulations.

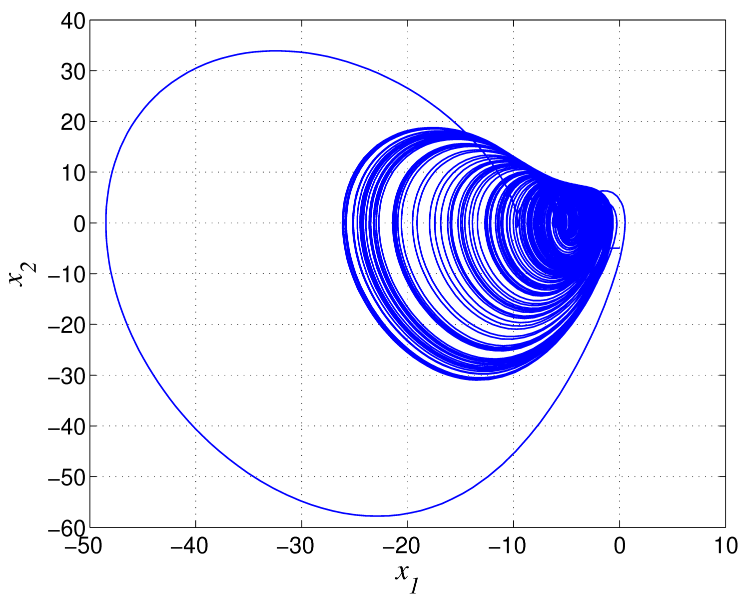

Figure 1: Phase portrait of the chaotic non-equilibrium hyperjerk system on (x_1,x_2) plane for $(a,b,c) = (2.2,5,2)$ and $X(0) = (0,-5,0,0)$

Figures 1–3 illustrate the two-dimensional projections of the chaotic non-equilibrium hyperjerk system (7) on (x_1,x_2), (x_1,x_3), (x_1,x_4) planes respectively.

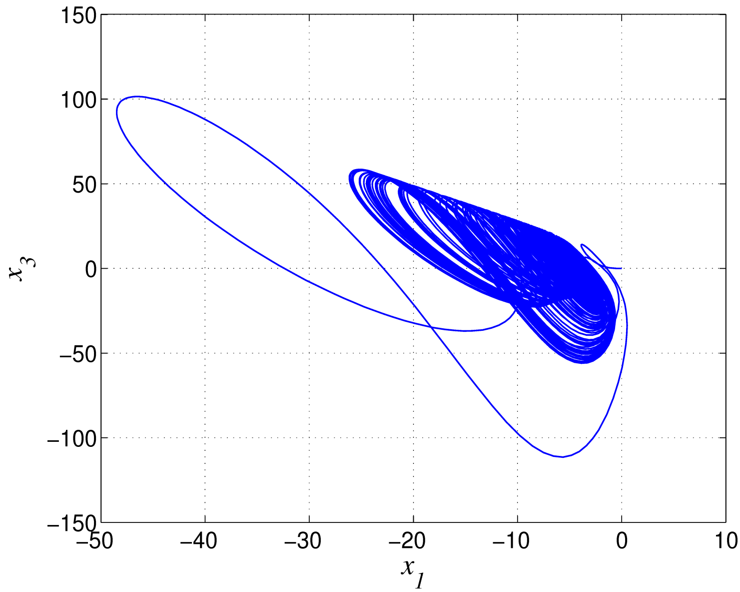

Figure 2: Phase portrait of the chaotic non-equilibrium hyperjerk system on (x_1,x_3) plane for $(a,b,c) = (2.2,5,2)$ and $X(0) = (0,-5,0,0)$

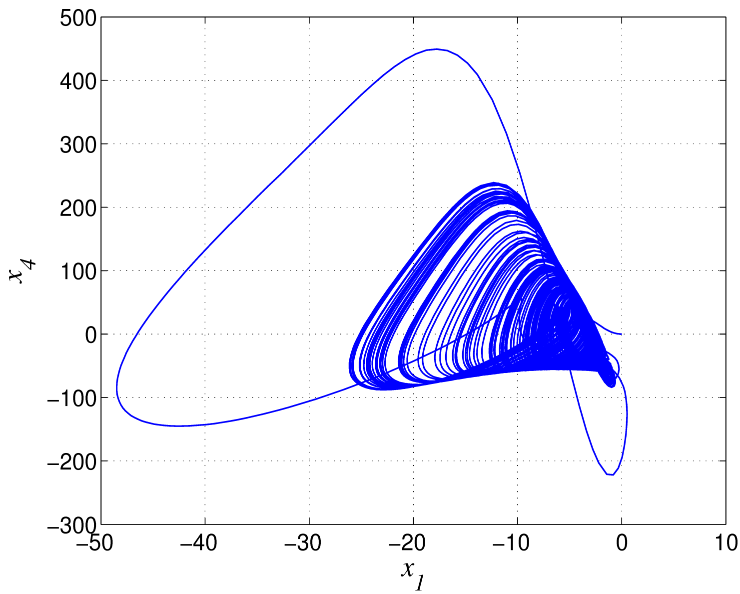

Figure 3: Phase portrait of the chaotic non-equilibrium hyperjerk system on (x_1,x_4) plane for $(a,b,c) = (2.2,5,2)$ and $X(0) = (0,-5,0,0)$

3. Adaptive control of the new non-equilibrium hyperjerk system via backstepping control

In this section, we design an adaptive feedback control law for globally stabilizing the new non-equilibrium hyperjerk system via backstepping control method.

We take the new non-equilibrium hyperjerk system with a single feedback control given by

$$\begin{cases} \dot{x}_1 = x_2\,, \\ \dot{x}_2 = x_3\,, \\ \dot{x}_3 = x_4\,, \\ \dot{x}_4 = ax_4 + bx_1x_2 + x_1x_4 + c(x_1^2 + x_1x_3 + 1) + u\,, \end{cases} \tag{11}$$

where a, b, c are unknown parameters and u is an adaptive feedback control law which uses a time-varying parameter estimate $(A(t), B(t), C(t))$ in lieu of (a, b, c).

The parameter estimation errors are defined as:

$$e_a(t) = a - A(t), \quad e_b(t) = b - B(t), \quad e_c(t) = c - C(t). \tag{12}$$

It is easy to see that

$$\dot{e}_a = \dot{A}, \quad \dot{e}_b = \dot{B}, \quad \dot{e}_c = \dot{C}. \tag{13}$$

Next, we establish the main adaptive control result of this section.

Theorem 1 *The new non-equilibrium hyperjerk system (11) with unknown parameters (a, b, c) is globally and asymptotically stabilized by the adaptive feedback control law,*

$$\begin{aligned} u(t) = & -5x_1 - 10x_2 - 9x_3 - [A(t) + 4]x_4 - B(t)x_1x_2 - x_1x_4 \\ & - C(t)(x_1^2 + x_1x_3 + 1) - Kz_4\,, \end{aligned} \tag{14}$$

where K is a positive constant,

$$z_4 = 3x_1 + 5x_2 + 3x_3 + x_4 \tag{15}$$

and the update law for the parameter estimates $(A(t), B(t), C(t))$ is given by

$$\begin{cases} \dot{A}(t) = z_4x_4\,, \\ \dot{B}(t) = z_4x_1x_2\,, \\ \dot{C}(t) = z_4(x_1^2 + x_1x_3 + 1). \end{cases} \tag{16}$$

Proof. We prove this result via Lyapunov stability theory [44].

In backstepping control method, we start with a quadratic Lyapunov function

$$V_1(z_1) = \frac{1}{2} z_1^2, \tag{17}$$

where

$$z_1 = x_1\,. \tag{18}$$

Differentiating V_1 along (11), we find that

$$\dot{V}_1 = z_1 \dot{z}_1 = x_1 x_2 = -z_1^2 + z_1(x_1 + x_2). \tag{19}$$

We set

$$z_2 = x_1 + x_2\,. \tag{20}$$

The equation (19) can be simplified as

$$\dot{V}_1 = -z_1^2 + z_1 z_2\,. \tag{21}$$

Next, we take a quadratic Lyapunov function

$$V_2(z_1, z_2) = V_1(z_1) + \frac{1}{2} z_2^2 = \frac{1}{2}\left(z_1^2 + z_2^2\right). \tag{22}$$

Differentiating V_2 along (11), we find that

$$\dot{V}_2 = -z_1^2 - z_2^2 + z_2(2x_1 + 2x_2 + x_3). \tag{23}$$

We set

$$z_3 = 2x_1 + 2x_2 + x_3\,. \tag{24}$$

Using (24), the equation (23) can be simplified as

$$\dot{V}_2 = -z_1^2 - z_2^2 + z_2 z_3\,. \tag{25}$$

Next, we take a quadratic Lyapunov function

$$V_3(z_1, z_2, z_3) = V_2(z_1, z_2) + \frac{1}{2} z_3^2 = \frac{1}{2}\left(z_1^2 + z_2^2 + z_3^2\right). \tag{26}$$

Differentiating V_3 along the dynamics (11), we find that

$$\dot{V}_3 = -z_1^2 - z_2^2 - z_3^2 + z_3(3x_1 + 5x_2 + 3x_3 + x_4). \tag{27}$$

We set

$$z_4 = 3x_1 + 5x_2 + 3x_3 + x_4\,. \tag{28}$$

Using (28), the equation (27) can be simplified as

$$\dot{V}_2 = -z_1^2 - z_2^2 - z_3^2 + z_3 z_4 \,. \tag{29}$$

Finally, we take the quadratic Lyapunov function

$$V(z_1,z_2,z_3,z_4,e_a,e_b,e_c) = V_3(z_1,z_2,z_3) + \frac{1}{2}z_4^2 + \frac{1}{2}e_a^2 + \frac{1}{2}e_b^2 + \frac{1}{2}e_c^2. \tag{30}$$

It is clear that V is a positive definite function on $\mathbf{R}^7$.
Differentiating V along (11), we get

$$\dot{V} = -z_1^2 - z_2^2 - z_3^2 - z_4^2 + z_4(z_4 + z_3 + \dot{z}_4) - e_a\dot{A} - e_b\dot{B} - e_c\dot{C}. \tag{31}$$

The equation (31) can be expressed in a compact manner as

$$\dot{V} = -z_1^2 - z_2^2 - z_3^2 - z_4^2 + z_4 S - e_a\dot{A} - e_b\dot{B} - e_c\dot{C}, \tag{32}$$

where

$$S = z_4 + z_3 + \dot{z}_4 = z_4 + z_3 + 3\dot{x}_1 + 5\dot{x}_2 + 3\dot{x}_3 + \dot{x}_4 \,. \tag{33}$$

It is easy to see that

$$\begin{aligned} S = {} & 5x_1 + 10x_2 + 9x_3 + (a+4)x_4 + bx_1x_2 + x_1x_4 \\ & + \; c(x_1^2 + x_1x_3 + 1) + u. \end{aligned} \tag{34}$$

Substitution of the adaptive control law (14) into (34) yields the result

$$S = [a - A(t)]x_4 + [b - B(t)]x_1x_2 + [c - C(t)](x_1^2 + x_1x_3 + 1) - Kz_4 \,. \tag{35}$$

Using (13), it is easy to simplify (35) as

$$S = e_a x_4 + e_b x_1 x_2 + e_c(x_1^2 + x_1x_3 + 1) - Kz_4 \,. \tag{36}$$

Combining (36) and (32), we find that we obtain

$$\begin{aligned} \dot{V} = {} & -z_1^2 - z_2^2 - z_3^2 - (1+K)z_4^2 + e_a(z_4x_4 - \dot{A}) \\ & + \; e_b(z_4x_1x_2 - \dot{B}) + e_c[z_4(x_1^2 + x_1x_3 + 1) - \dot{C}]. \end{aligned} \tag{37}$$

Substituting the update law (16) into (37), we get

$$\dot{V} = -z_1^2 - z_2^2 - z_3^2 - (1+K)z_4^2, \tag{38}$$

which is a negative semi-definite function on $\mathbf{R}^7$.

Using Barbalat's lemma [44], we conclude that $\mathbf{z}(t) \to \mathbf{0}$ asymptotically as $t \to \infty$ for all initial conditions $\mathbf{z}(0) \in \mathbf{R}^4$.

Hence, it is immediate that $\mathbf{x}(t) \to \mathbf{0}$ asymptotically as $t \to \infty$ for all initial conditions $\mathbf{x}(0) \in \mathbf{R}^4$.

This completes the proof. □

For numerical simulations, we suppose that the parameter values of the new hyperjerk system (11) are taken as in the chaotic case, *i.e.* $(a, b, c) = (2.2, 5, 2)$. Also, we take $K = 20$.

As initial conditions of the new hyperjerk system (11), we take $x_1(0) = 2.5$, $x_2(0) = 1.2$, $x_3(0) = 3.6$ and $x_4(0) = 1.7$.

Also, as initial conditions of the parameter estimates, we take $A(0) = 4.3$, $B(0) = 10.4$ and $C(0) = 8.7$.

In Figure 4, the asymptotic convergence of the controlled state $\mathbf{x}(t)$ is exhibited, when the controls (14) and (16) are implemented.

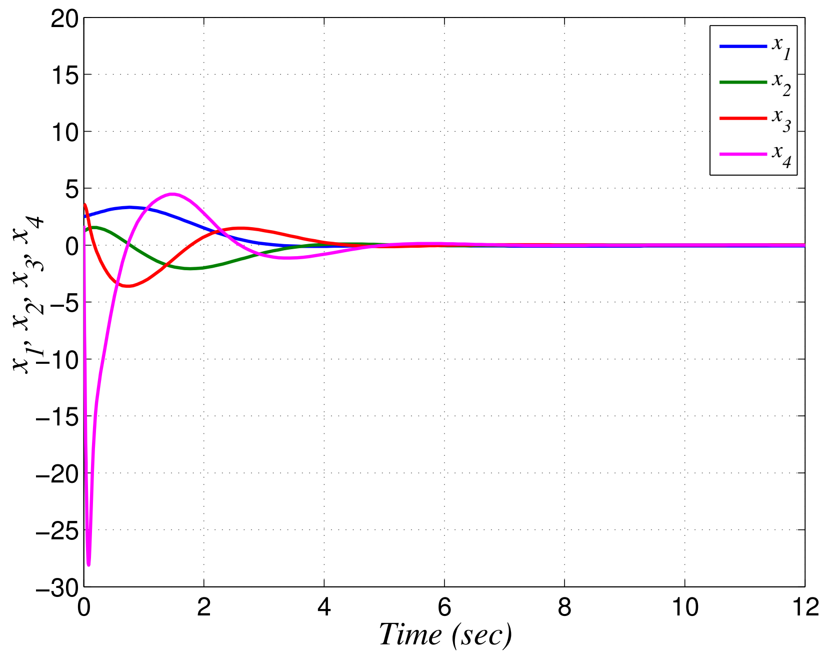

Figure 4: Time-history of the controlled states of the hyperjerk system (11)

4. Circuit implementation of the non-equilibrium hyperjerk system

In this section, we present an electronic circuit realizing new hyperjerk system (7) to verify its feasibility. The designed circuit is shown in Figure 5. Here we have used the general known approach [39–42], in which the state variables of system (7) correspond to the voltages at operational amplifiers U_1, U_2, U_3, and U_4, respectively.

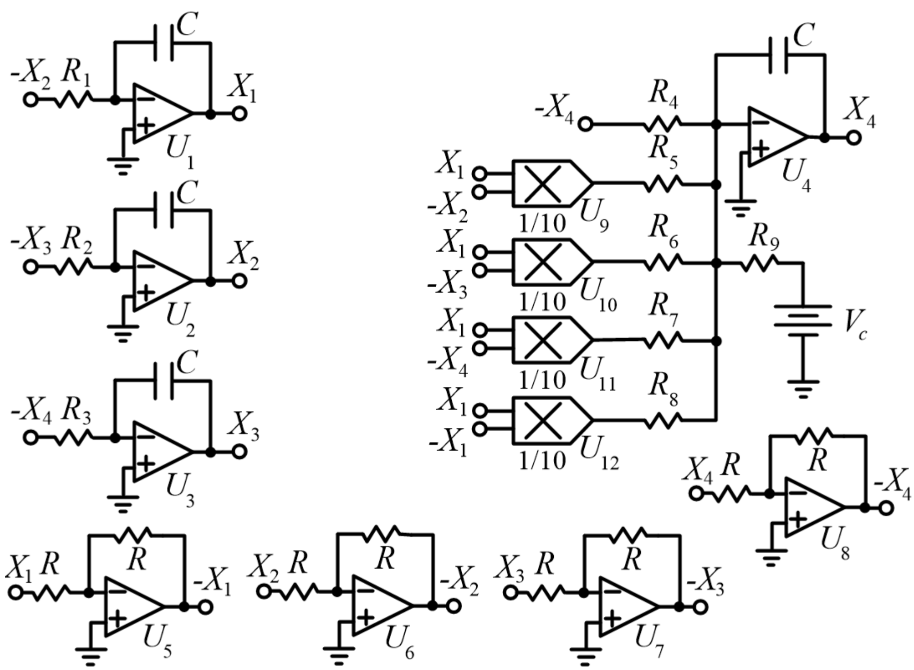

Figure 5: Designed circuit for the non-equilibrium hyperjerk system (7)

By applying Kirchhoff's laws to the designed electronic circuit, we have equations of circuit

$$\begin{cases} \dot{X}_1 = \dfrac{1}{R_1C}X_2\,, \\ \dot{X}_2 = \dfrac{1}{R_2C}X_3\,, \\ \dot{X}_3 = \dfrac{1}{R_3C}X_4\,, \\ \dot{X}_4 = \dfrac{1}{R_4C}X_4 + \dfrac{1}{10R_5C}X_1X_2 + \dfrac{1}{10R_6C}X_1X_3 \\ \qquad + \dfrac{1}{10R_7C}X_1X_4 + \dfrac{1}{10R_8C}X_1^2 - \dfrac{1}{R_9C}V_c\,, \end{cases} \tag{39}$$

where X_1, X_2, X_3, and X_4 are the voltages at operational amplifiers U_1, U_2, U_3, and U_4. It is simple to see that Eq. (39) matches with Eq. (7).

The circuit is implemented in PSpice with $C = 1$ nF, $R_1 = R_2 = R = 100$ kΩ, $R_3 = 20$ kΩ, $R_4 = 45.45$ kΩ, $R_5 = R_7 = 1$ kΩ, $R_6 = R_8 = 2.5$ kΩ, $R_9 = 2.5$ MΩ, and $V_c = -1\ V_{DC}$.

Figures 6, 7, and 8 illustrate phase portraits of the circuit (39) in (X_1, X_2), (X_1, X_3) and (X_1, X_4) coordinate planes respectively.

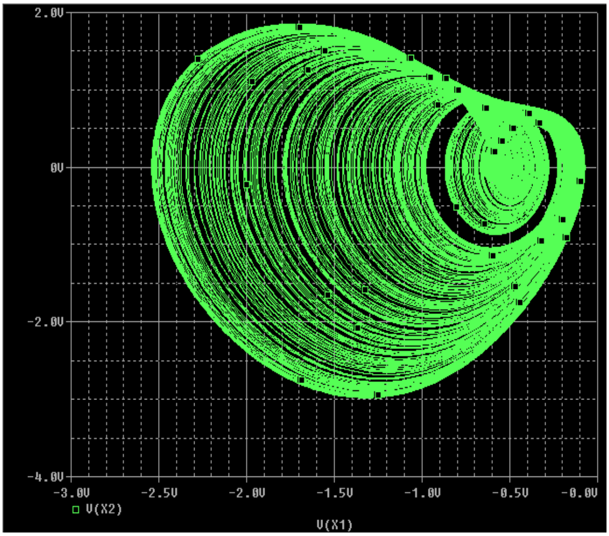

Figure 6: Phase portrait of the designed circuit (39) in (X_1, X_2)-plane

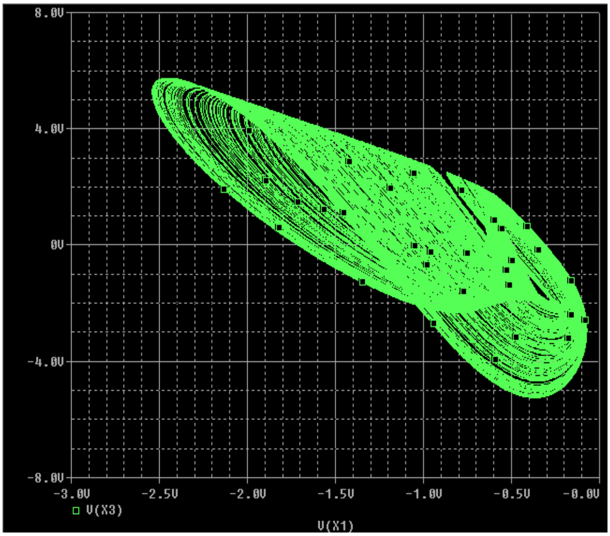

Figure 7: Phase portrait of the designed circuit (39) in (X_1, X_3)-plane

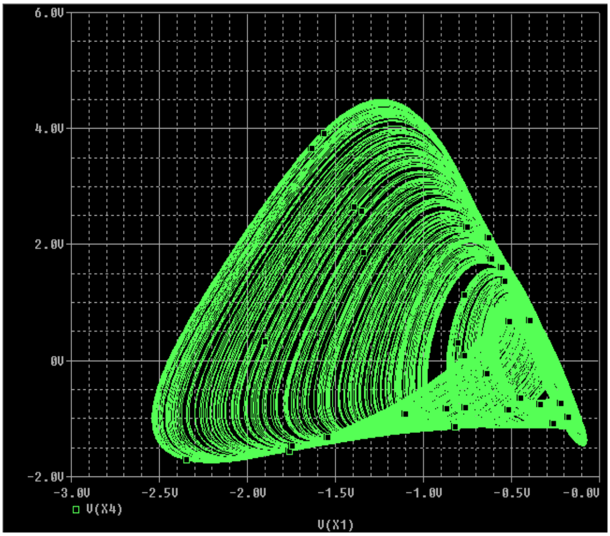

Figure 8: Phase portrait of the designed circuit (39) in (X_1, X_4)-plane

We notice that circuit simulations of the new non-equilibrium hyperjerk system (39) shown in Figures 6–8 show good agreement with the MATLAB simulations of the non-equilibrium hyperjerk system (7) shown in Figures 1–4. This validates the theoretical model of the non-equilibrium hyperjerk system.

5. Conclusion

We presented results for a new 4-D chaotic hyperjerk system with four quadratic nonlinearities. An interesting feature of this new chaotic hyperjerk system is that it has no equilibrium points and hence it possesses a hidden chaotic attractor. We verified chaos in the new hyperjerk system by a positive Lyapunov exponent. Adaptive backstepping controller was designed for the global chaos control of the non-equilibrium hyperjerk system with a hidden attractor and MATLAB simulation was given in detail. An electronic circuit for realizing the non-equilibrium hyperjerk system was implemented in PSpice and circuit simulations were detailed.

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