

A mathematical model for calculating contact mass loss resulting from bridge disintegration in vacuum

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Abstract: The paper presents descriptions of bridge disintegration types and contact mass loss in the bridge stage. There is presented Matlab solvers to solve equation describing dynamic changes of temperature in the bridge region. The final result of program calculations is the mass loss and the volume of the metal of contacts which was lost during the bridge stage.

Key words: electrical arc, contact mass loss, contact bridge, software

1. Introduction

Phenomena which occur during current breaking in an electric circuit with the use of a contact switch mainly depend on thermal-erosion processes, but also on clamp force (contact force) and movement parameters. The understanding of thermal-erosion processes in low-voltage power switches at switching currents under short-circuit conditions is of great theoretical and practical importance. These processes result in contact erosion and determine the basic ratings of switching devices. The intensity of thermal-erosion processes determines contact life of a switch. Figure 1 presents the basic characteristics and phenomena that affect thermal-erosion processes in contacts [1-3].

In the process of breaking electric current there can be differentiated four stages: a bridge stage, a transient stage, an arc stage and a 'post-arc' stage.

The bridge stage includes phenomena that occur during transient contact from the moment when contacts start to separate until the contacts are completely separated. The transient stage includes phenomena that occur from the moment of bridge disintegration until an arc structure is created. The arc stage includes the phenomenon of arc burning until the moment of extinguishing, whereas the 'post-arc' stage – the phenomena of increasing electric strength of the contact gap after arc extinguishing. During each of these stages there can occur contact mass loss.

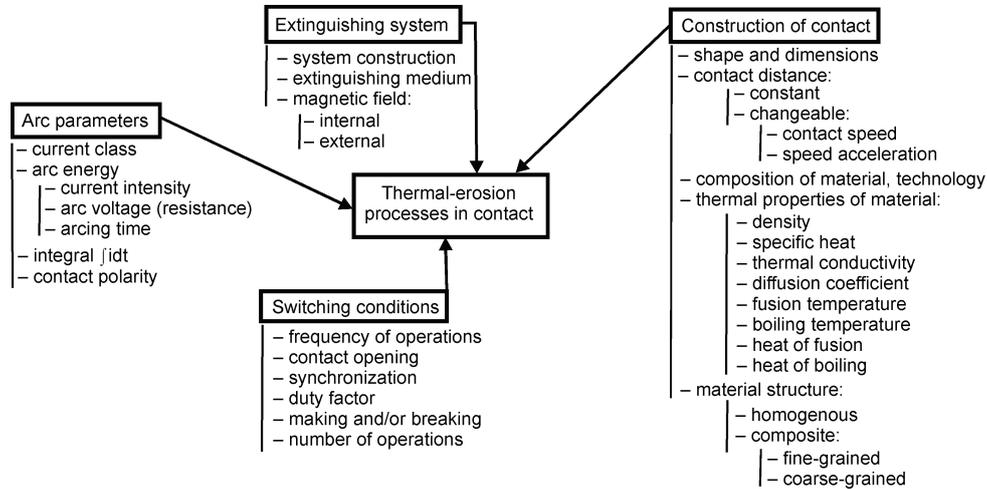


Fig. 1. Main parameters and factors that affect thermal processes in a contact [3]

The paper focuses in more detail on contact mass loss only in the bridge stage in vacuum.

2. A general equation of contact mass balance

During each stage of the process of breaking electric current in a contact switch there occurs contact mass loss as a result of, among other things, migration of material from one contact to the other. The amount of migrating material depends on the contact material and the electrical characteristics of an arc and conducted current. Moreover, an electric arc causes oxidation of contacts. In a short arc there occurs oxidation of anode surface which is penetrated by fast primary electrons. In a long arc, ionization causes oxidation of cathode surface.

It is regarded that weight loss or volume loss of contact mass occurs when contact material is removed in the form of charged particles, metal vapors, melted material and material in the solid state [4]. The condition is that equally high energy is fed to the electrodes and the temperature of fusion and boiling of metals from which the contact composite material is made is reached on the surface of contacts.

Figure 2 presents a model of material migration in a short arc in a high-current contact at lower currents. Figure 3 presents a model of material migration under arcing conditions at high currents. It is assumed here that the diameter of the arc root is equal to the diameter of the contact.

In both cases the equation of contact mass balance during the occurrence of an electric arc in a contact can be written generally in the following way:

- for the anode

$$\Delta m_{Aob} = \Delta m_{A_1} + \Delta m_{A_2} + \Delta m_{A_3}, \quad (1)$$

- for the cathode

$$\Delta m_{C_{ob}} = \Delta m_{C_1} + \Delta m_{C_2} + \Delta m_{C_3}, \quad (2)$$

where: $\Delta m_{A_{ob}}$ and $\Delta m_{C_{ob}}$ – the calculated mass of melted and evaporating anode and cathode material, Δm_{A_1} and Δm_{C_1} – the mass dispersed into the contact surroundings in the form of vapors and drops from the anode and the cathode, Δm_{A_2} and Δm_{C_2} – the mass of anode and cathode material migrating in the form of a stream of metal vapors and, possibly, drops onto the opposite electrode (the so-called migration of material from one electrode onto the other), Δm_{A_3} and Δm_{C_3} – the mass which is transported or sediments again in the anode and cathode spot region or outside it in the form of drops, streams or a ‘crown’.

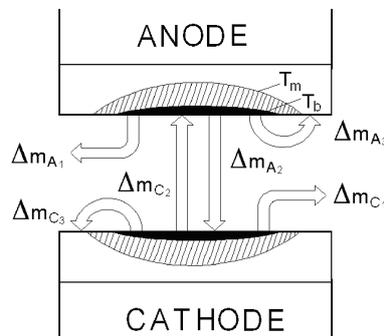


Fig. 2. A schematic representation of contact mass balance components in a short arc at lower currents [5]

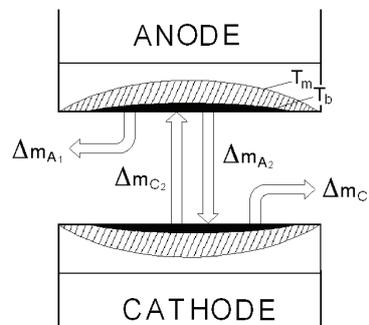


Fig. 3. A schematic representation of contact mass balance components in a short arc at high currents [1]

Weight loss of contact mass measured for the anode and the cathode:

$$\Delta m_{A_p} = \Delta m_{A_1} + (\Delta m_{A_2} - \Delta m_{C_2}), \quad (3)$$

$$\Delta m_{C_p} = \Delta m_{C_1} + (\Delta m_{C_2} - \Delta m_{A_2}). \quad (4)$$

The proportion of contact mass migration in the form of plasma streams can be so huge in the overall balance that instead of a loss there can occur an increase of contact mass, which sometimes takes place in the case of the cathode when:

$$(\Delta m_{C_2} - \Delta m_{A_2}) > \Delta m_{C_1} \quad (5)$$

and then Δm_{C_p} has a minus value.

The equations of contact mass balance (3) and (4) for high currents take the following form:

$$\Delta m_{A_{ob}} = \Delta m_{A_1} + \Delta m_{A_2}, \quad (6)$$

$$\Delta m_{C_{ob}} = \Delta m_{C_1} + \Delta m_{C_2}. \quad (7)$$

Equations (6) and (7) also express the measured mass loss. However, it should be noted that in this case Δm_{A_1} and Δm_{C_1} are considerably larger, as they are enlarged by elements Δm_{A_3} and Δm_{C_3} .

3. Bridge disintegration types

Arc bridge may disintegrate as a result of different phenomena. Generally, there can be differentiated the following types of bridge disintegration [6]:

- M – mechanical – when the bridge does not reach the temperature of boiling and is mechanically broken as contacts separate;
- T – thermal – when the bridge disintegrates as its spatial structure is broken as a result of internal thermodynamic processes;
- W – vibration – when the bridge is broken as a result of generation of vibrations of an increasing amplitude.

There can be distinguished two basic types of mechanical disintegration:

- MS – mechanical disintegration of a solid bridge, when temperature in the bridge does not exceed the temperature of fusion;
- MC – mechanical disintegration of a liquid bridge, when the temperature of fusion is exceeded.

As presented below, there are also different types of thermal disintegration:

- TM – thermomechanical;
- TN – thermal normal bridge disintegration as a result of boiling;
- TA – thermal abnormal bridge disintegration – after its transition into a supercritical state as a result of exceeding, by an increasing plasma pressure, the value of electrodynamic pressure.

W disintegration does not occur independently. Vibrations can only appear after the fusion of a bridge in the range of currents corresponding to T-types of disintegration. That is why there are distinguished mixed types of TW: TWN, when vibrations accompany disintegration of a TN type and TWA, when vibrations occur during TA disintegration.

A summary of all types of bridge disintegration and their thermodynamic criteria is given in Figure 4 and in Table 1.

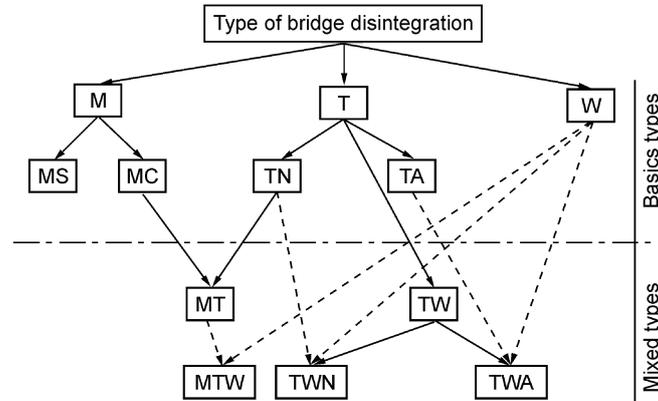


Fig. 4. A classification of bridge disintegration types

Table 1. Bridge disintegration types and their criteria

Bridge disintegration type	Thermodynamic criteria	Voltage criteria	Current range	Remarks
MS	$T_m < T_{to}$ $p_m \approx p_o$	$u_{mm} < u_{to}$	Very low currents	–
MC	$T_{to} \leq T_m \leq T_{wo}$ $p_m \approx p_o$	$u_{to} \leq u_{mm} \leq u_{wo}$	Low currents	–
MT	$T_m \approx T_{wo}$ $p_m \approx p_o$	$u_{mm} \approx u_{wo}$	Medium currents	–
TN	$T_m = T_{wp}$ $T_{wo} \leq T_{wp} \leq T_k$	$u_{mm} = u_{wp}$ $u_{wo} < u_{wp} \leq u_{wk}$	High currents	TWN – u_m voltage oscillations 1) static bridge 2) dynamic bridge
TA	$T_m > T_k$	$u_{mm} > u_{wk}$	Very high currents	TWA – u_m voltage oscillations
TWA	$p_m > p_k$			

A description of quantities used in Table 1: T_m, p_m – temperature and pressure in the bridge; T_k, p_k – critical temperature and pressure; p_o – pressure of the surroundings; T_t, T_w – temperature of fusion and boiling of the metal of the bridge; T_{to}, T_{wo} – temperature of fusion and boiling of the metal of the bridge at pressure p_o ; u_t, u_w, u_{wk} – voltage of fusion, voltage of boiling and critical voltage; u_{mm} – bridge disintegration voltage; u_{to}, u_{wo} – voltage of fusion and boiling of the metal of the bridge at pressure p_o ; T_{wp}, u_{wp} – temperature and voltage of boiling at pressure p_m .

4. Contact mass loss in the bridge stage

Contact mass loss resulting from metallic bridge disintegration occurs both during mechanical and thermal bridge disintegration. At the moment of completion of the bridge stage there

takes place volume evaporation of the metal of contacts. The volume of evaporation is limited by an isotherm of the temperature of boiling of the given metal in vacuum. Contact mass loss that occurs as a result of bridge explosion can be determined experimentally on the basis of the volume of a crater that remains after the bridge and the density of a metal in a steady state of current flow, i.e. at normal pressure. It is justified, as increased volume of the bridge during the opening of contacts results only from thermal expansion of the metal of contacts.

Independently of the bridge disintegration type, removal of contact material occurs as a result of volume evaporation. On the basis of bridge disintegration time and the isotherm of boiling of the metal it is possible to determine analytically the volume of the metal ejected during a bridge explosion, whereas mass loss can be determined on the basis of volume and density. As a result of a bridge explosion, from both contacts there evaporates material of the volume of a sphere sector V_e (Fig. 5), which is expressed by the following formula:

$$V_e = \frac{\pi h_e^2}{3} (3R_{WP} - h_e), \tag{8}$$

where: h_e – height of the sphere sector, R_{WP} – radius of the isotherm of temperature T_{WP} .

The volume of material that erodes from both contacts equals:

$$V_e = \frac{\pi}{3} \left(2R_{WP}^3 - \frac{3}{4} at_m^2 R_{WP}^2 + \frac{a^3 t_m^6}{64} \right), \tag{9}$$

where: t_m – bridge disintegration time, $\frac{at_m^2}{2}$ – mutual distance of contacts at time t_m .

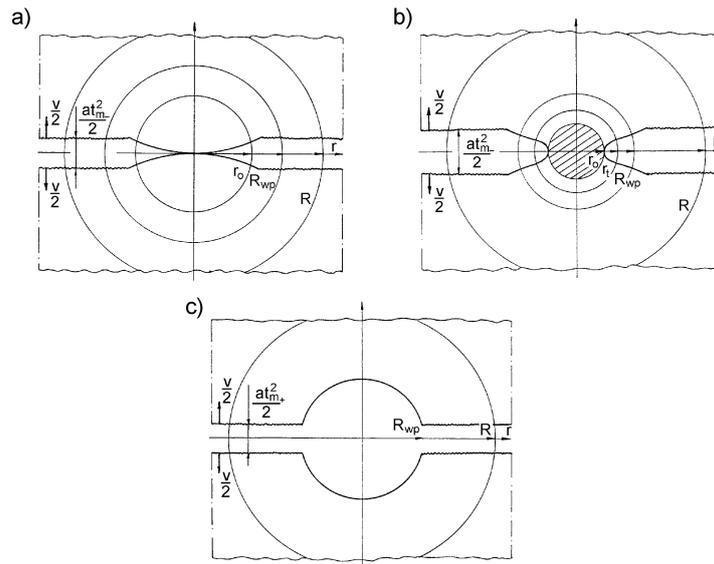


Fig. 5. A simplified distribution of a bridge temperature field [6]: a) before thermal disintegration, b) before mechanical disintegration, c) after a bridge explosion

In order to be able to calculate mass loss in the bridge stage it is first necessary to determine temperature distribution in the bridge region in a steady state of current flow and then during the opening of contacts (Fig. 6). On the basis of the temperature of contacts there are determined other physical quantities dependent on temperature, e.g. thermodynamic (internal) pressure. The bridge stage is completed only if internal and electromagnetic pressure become equal (pinch effect). Initially, internal pressure is low due to relatively low temperature in a contact, whereas electrodynamic pressure is high and dependent on the current conducted through the contacts. As the contacts separate, the actual cross-section surface of a metallic bridge becomes smaller, which results in a rise of electromagnetic pressure. As a result of the bridge cross-section surface becoming smaller, current density increases, which results in a rapid rise of both temperature and internal pressure. Having exceeded the value of electrodynamic pressure, internal pressure causes a bridge explosion.

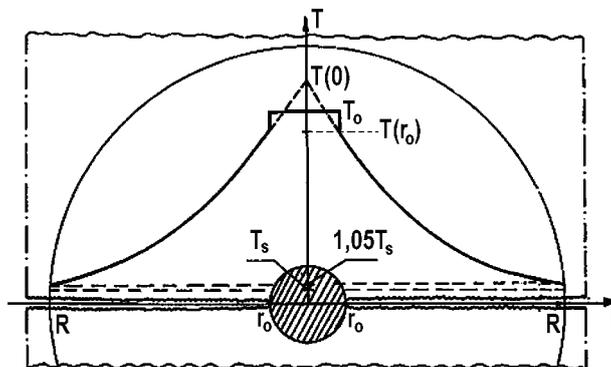


Fig. 6. A course of temperature depending on the distance from the centre of symmetry of the field – a course determined for a steady state of current flow [6]

5. A differential equation describing dynamic changes of temperature in the bridge region

A difference equation which describes dynamic changes of temperature in a bridge is a parabolic equation of a general form described by formula (10). The analyzed model is in a spherical coordinate system and there is assumed a radial temperature distribution against the centre of the coordinate system located in the centre of an equivalent contact area. In such a case, the number of spatial variables in the equation is reduced to one and time. The remaining elements of the equation are the functions of temperature: a temperature compensation coefficient κ_s , volume density of power of the heat source $g(t)$ and thermal conductivity λ_s .

The equation describes two areas. The first one is the internal region of the equivalent contact area in which there is heat input determined by the density of power of the heat source, whereas the other one is the region from r_0 to R in which the heat source does not operate. Radius R determines the boundary of the analyzed area, on which there is determined a boundary condition of equation (10) in the form of (11), i.e. on the boundary $r = R$ of the analyzed

area it assumes the temperature of the surroundings. All calculations were performed in Matlab for copper material [7].

$$\kappa_s(T) \left[\frac{\delta^2 T}{\delta r^2} + \frac{2}{r} \frac{\delta T}{\delta r} \right] = \frac{\partial T}{\partial t} - \frac{g(t)}{\lambda_s(T)} \kappa_s(T), \quad (10)$$

where volume density of the heat source $g(t)$ assumes: $g(t) = 0$ for $r > r_0(0)$ and $g(t) = g_0$ for $r \leq r_0(0)$.

Boundary conditions:

$$\lim_{r \rightarrow 0} \kappa_s(T_s) r^2 \frac{\delta T}{\delta r} = 0; \quad (11)$$

$$T(R) = T_{ot}.$$

The initial condition:

$T(r, 0)$ – determined in calculations for a steady state of current flow; $\kappa_s(T_s)$ – a compensation coefficient of temperatures of the solid phase of the metal of contacts.

$$\kappa_s(T_s) = - \frac{\lambda_s(T_s)}{c_s(T_s) \gamma_s(T_s)}, \quad (12)$$

$\lambda_s(T_s) = 395 - 0.109T_s + 2.81 \cdot 10^{-5}T_s^2$ [W/(m · K)] – thermal conductivity of the solid phase of the metal of contacts, $c_s(T_s) = 358.4 + 9.6 \cdot 10^{-2}(T_s + 273)$ [J/(kg · K)] – specific heat, $\gamma_s(T_s) = 8.93 \cdot 10^3 - 0.56T_s$ [kg/m³] – density of the solid phase of the metal of contacts.

Figure 7 presents an example of a temperature distribution calculated in 3D in a steady state of current flow through the contact.

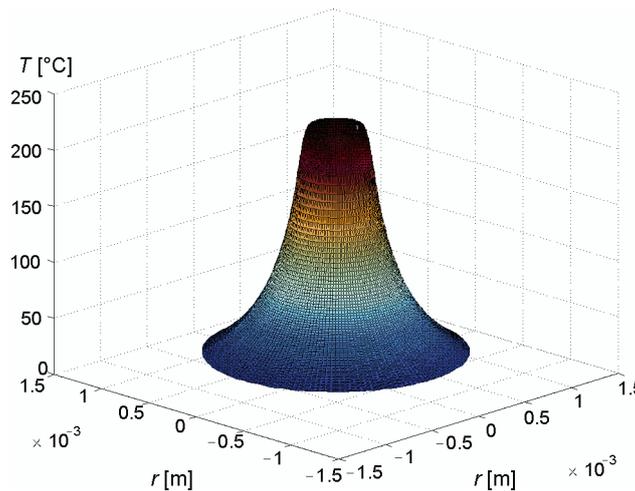


Fig. 7. A three-dimensional temperature distribution in a steady state of current flow through the contact for $F = 100$ N, $I = 4$ kA, copper contacts of 10 mm diameter

To calculate this temperature (for a steady and dynamic state), we need to know a lot of parameters. Some of them will be shown below.

Equivalent radius of the actual contact area (13)

$$r_0(0) = \sqrt{\frac{F}{\pi \xi H_B(T_{ot})}}, \quad (13)$$

where: F – contact force, $H_B(T_{ot})$ – Brinell hardness at ambient temperature; ξ – elastic deformation coefficient.

Power density heat sources operating in equivalent radius of the actual contact area (14)

$$g_0 = \frac{I^2 \rho_s(T_{ot})}{2V_0 r_0(T_{ot})}, \quad (14)$$

where: I – contact current at the steady state; $\rho_s(T_{ot})$ – density at ambient temperature; V_0 – volume determined by replacing of contact radius sphere; $r_0(T_{ot})$ – equivalent radius of the actual contact area at the steady state.

The initial stage pressure of contact bridge (15)

$$p_{H0} = \frac{\mu}{6\pi^2} \frac{I^2}{[r_0(0)]^2}, \quad (15)$$

where: μ – magnetic permeability of the contact material; I – contact current at the steady state; $r_0(0)$ – equivalent radius of the actual contact area for $t = 0$.

Brinell hardness determined for the metal contacts in the solid phase the following formula (16):

$$H_B(T) = 4.22 \cdot 10^7 - 43.1T + 2.5 \cdot 10^{-2}(T)^2 \text{ [kg/m}^2\text{]}. \quad (16)$$

6. User interface

The program for calculating contact mass loss resulting from bridge disintegration has a simple interface (Fig. 8) that makes it possible to enter physical input data and characteristics of a contact material, and also it allows to run calculations using script s3MassLoss.m.

Parameters entered into the user interface are saved into a text file paramWe.mat and then read by the main calculating program. After completing calculations, the program displays their results and charts (e.g. the radius of an equivalent contact area, a temperature distribution in a steady and dynamic state, electrodynamic pressure) in the main MatLab window in separate windows.

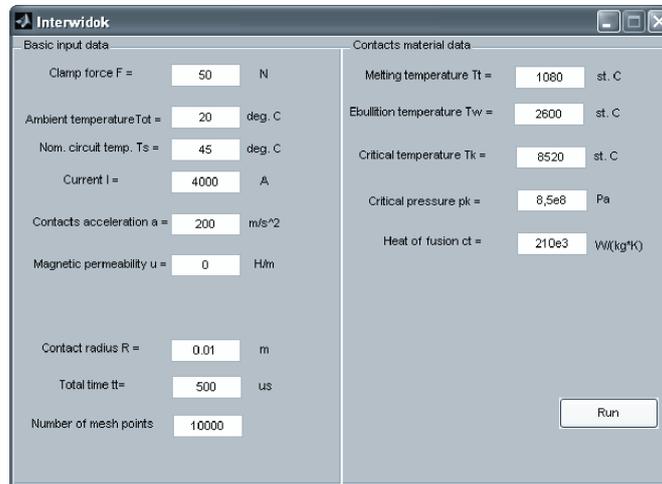


Fig. 8. A view of the user interface window

7. The results of calculations

As it has already been mentioned, the essential element in the application for calculating contact mass loss is the solution of differential equations describing temperature distribution in the dynamic state. Figures 9-13 present temperature distribution charts for a steady state of current flow and for a dynamic state of contact opening for different input parameters, i.e. clamp force, current value and acceleration of the separation of contacts. The results refer to copper contacts of a 10 mm intersection radius.

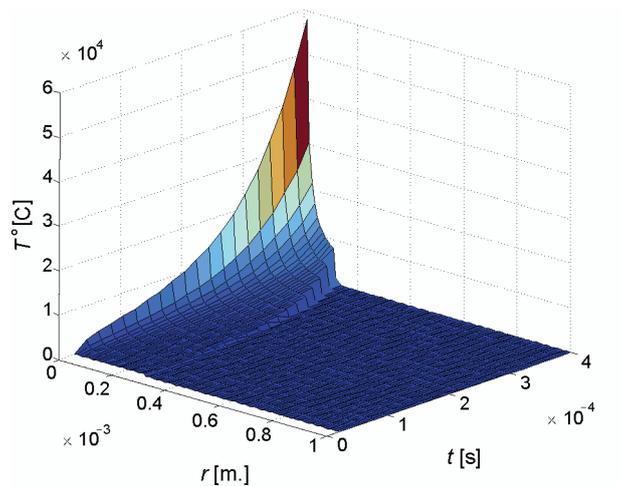


Fig. 9. A dynamic temperature distribution for $F = 50$ N, $I = 4000$ A, $a = 10$ m/s²

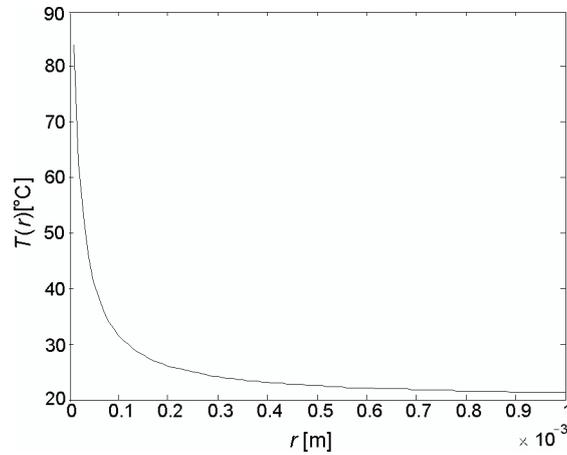


Fig. 10. A temperature distribution in a steady state for $F = 100$ N, $I = 3000$ A, $a = 10$ m/s²

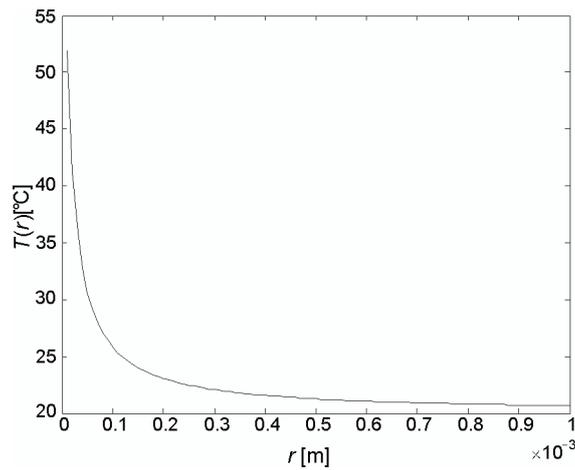


Fig. 11. A temperature distribution in a steady state for $F = 200$ N, $I = 3000$ A, $a = 10$ m/s²

The results of calculations of volume and contact mass loss for different input parameters for a copper contact of a 10 mm radius are presented in Table 2.

We calculated here the volume loss and mass loss that are to be caused only by bridge disintegration.

Contact mass loss resulting from bridge disintegration during breaking even a high current is relatively small. The order of magnitude of bridge contact mass loss are micrograms. In the literature there have so far been no theoretical considerations on contact erosion in the bridge stage in electromagnetic switches (power switches, circuit breakers). The proposed algorithm is very useful in cases when an electric arc lasts for a very short time in comparison with the duration of the bridge stage. Such a case occurs in low-voltage rating (e.g. 24 V, 60 V or

110 V) DC power switches used on ships or in arc furnaces used in aluminum plants. The bridge phase in this type of switches may be considerably important.

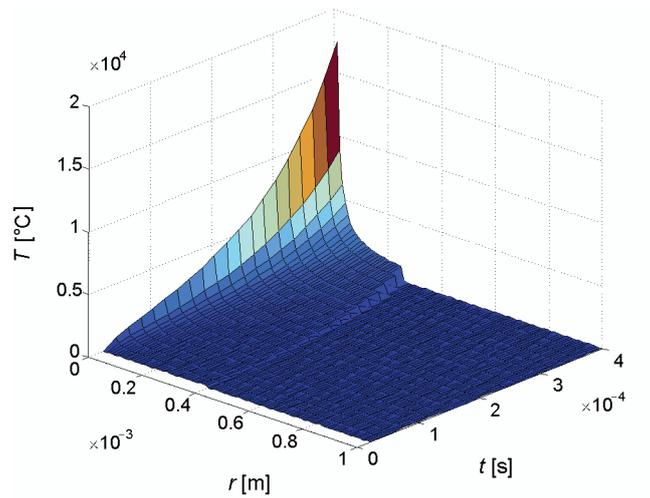


Fig. 12. A dynamic temperature distribution for $F = 100$ N, $I = 3000$ A, $a = 10$ m/s²

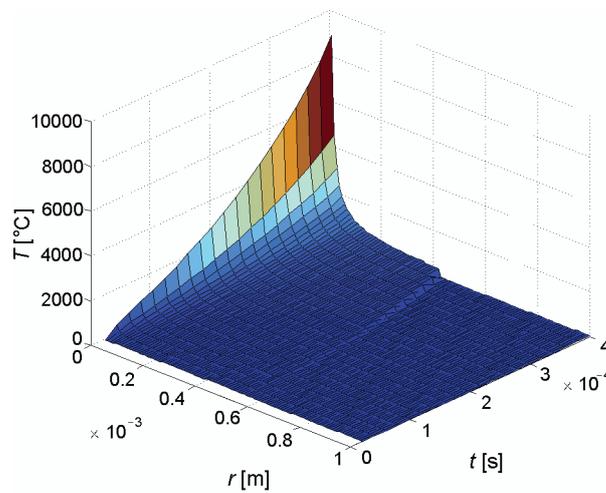


Fig. 13. A dynamic temperature distribution for $F = 200$ N, $I = 3000$ A, $a = 10$ m/s²

8. Conclusions

The program for calculating contact mass loss resulting from bridge disintegration includes mathematical analysis of phenomena that occur during the bridge stage. The final result of

program calculations is the mass loss and the volume of the metal of contacts which was lost during the bridge stage.

Table 2. The results of calculations of mass loss and volume for copper contacts of a 10 mm radius

Clamp force F [N]	Current I [A]	Acceleration a [m/s ²]	Volume loss V_e [m ³]	Mass loss M_e [kg]
50	4000	10	$1.066 \cdot 10^{-12}$	$9.249 \cdot 10^{-9}$
50	4000	50	$8.211 \cdot 10^{-13}$	$8.954 \cdot 10^{-9}$
50	4000	100	$8.33 \cdot 10^{-14}$	$7.34 \cdot 10^{-10}$
100	4000	300	$2.757 \cdot 10^{-15}$	$2.448 \cdot 10^{-11}$
300	4000	300	$5.965 \cdot 10^{-15}$	$5.316 \cdot 10^{-11}$

During calculations it is possible to display a number of characteristics of physical quantities that describe phenomena both in the expansion phase and in the bridge phase, such as, for example, an equivalent radius of the actual contact area, volume density of the heat source, the characteristics of a contact material (Brinell hardness, density, electrical and thermal conductivity, thermal expansion) in the function of temperature, a temperature distribution at the time of a steady current flow, electromagnetic pressure and internal pressure in the function of time. The most important element in the program is the determination of a temperature distribution in the function of the contact radius and time during the opening of contacts. The variety of the displayed courses allows a user not only to obtain the result of mass loss in the form of a number, but also to analyze in more detail phenomena that occur in a contact and possibly to optimize input parameters for a chosen contact material.

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