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Controllability-Oriented Placement of Actuators for Active Noise-Vibration Control of Rectangular Plates Using a Memetic Algorithm

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For successful active control with a vibrating plate it is essential to appropriately place actuators. One of the most important criteria is to make the system controllable, so any control objectives can be achieved.

In this paper the controllability-oriented placement of actuators is undertaken. First, a theoretical model of a fully clamped rectangular plate is obtained. Optimization criterion based on maximization of controllability of the system is developed. The memetic algorithm is used to find the optimal solution. Obtained results are compared with those obtained by the evolutionary algorithm. The configuration is also validated experimentally.

Keywords: active control, flexible structures, actuators placement, controllability Gramian, evolutionary algorithm, memetic algorithm.

1. Introduction

Structural sound sources are acoustic radiators of increasing significance for active noise control (ANC) systems (Kozupa, Wiciak, 2010; Mazur, Pawełczyk, 2011; Pawełczyk, 2008). The problem of actuators placement on a vibrating structure has been a point of interest in recent years. Their effect on sound radiation has been analysed by different authors (Górski, Kozupa, 2012; Leniowska, 2009; Szemela *et al.*, 2012). Misplaced actuators may result in lack of controllability, which deteriorates the performance of the system. In many practical applications there is also a limit to the number of actuators, so they need to reach the best possible performance. Therefore, their locations have to be carefully chosen.

Different techniques have been proposed over the years. A survey of actuator placement in various engineering disciplines until 1999 is presented in (Padula, Kincaid, 1999). Later work in the area of actuator location on flexible structures is reviewed in (Frecker, 2003).

There are two basic approaches to optimize actuators placement. One approach is primarily focused on selecting a control strategy, defining a performance index, and then simultaneously determining both the optimal model-based controller and actuators placement. Performance of a linear quadratic regulator (LQR) controller was considered as an objective in (Kumar, Narayanan, 2007). The spatial H_2 norm of the closed-loop system was used as the performance index for a genetic algorithm in (Liu *et al.*, 2006). A computational method to design an H_∞ controller and corresponding optimal actuators locations was presented in (Arabyan, Chemishkian, 1998; Chemishkian, Arabyan, 1999). However, in such an approach, optimality of the obtained solution is dependent on the choice of a control strategy.

Another approach is based on an open-loop system analysis, and therefore it is independent on controller choice. A Gramian controllability was taken as an objective in (Sadri *et al.*, 1999; Han, Lee, 1999). Optimal placement of ten piezoelectric sensor/actuator pairs mounted on a cantilever plate using modified H_∞ norm was investigated in (Hale, Daraji, 2012). A controllability-oriented approach and spillover effect reduction was presented in (Pawełczyk, Wrona, 2013).

In this paper the controllability-oriented approach is adopted to solve the actuators placement problem. The proposed method is based on modeling the overall structure including position of actuators, and is totally independent of the control strategy. A fully-clamped isotropic rectangular plate is considered. A memetic algorithm (MA) is proposed to be applied to find efficient locations for actuators. MA method similarly to evolutionary algorithm (EA), is well adapted in finding the global optimal solution for a complicated problem such as the locations of actuators. However, MA is characterized by improved procedures for local search and can lead to a faster convergence and a statistically better solution. Optimization criterion used in this paper is based on the Gramian matrix.

2. Plate modeling

In this section, the overall state model of a plate with actuators bonded to its surface is derived. The modeling is based on the Rayleigh-Ritz assumed mode shape method. Fundamental issues of this theory are recalled below to set a reference for further reading.

According to the Kirchhoff-Love plate theory (Rao, 2007), the equation of motion in Cartesian coordinates is:

$$D\left(\frac{\partial^4 w}{\partial x^4}+2\frac{\partial^4 w}{\partial x^2\partial y^2}+\frac{\partial^4 w}{\partial y^4}\right)+m_s\frac{\partial^2 w}{\partial t^2}=f_a, \tag{1}$$

where

$$D=\frac{Eh^3}{12(1-\nu^2)}. \tag{2}$$

In (1) and (2) w is the plate transverse displacement; f_a is the total force generated by actuators; ∇^4 is the biharmonic differential operator; D is the flexural rigidity; E is the Young's modulus; ν is the Poisson's ratio; m_s is the mass per unit area of plate surface; and h is the plate thickness.

Considering only the transverse motion and neglecting the effect of rotary inertia, the kinetic energy of the plate T can be expressed as:

$$T=\frac{1}{2}\iint_S m_s\left(\frac{\partial w}{\partial t}\right)^2 \mathrm{d}x\,\mathrm{d}y, \tag{3}$$

where S is the surface of the plate. The strain energy U can be written as:

$$U=\frac{D}{2}\iint_S\left\{\left(\frac{\partial^2 w}{\partial x^2}\right)^2+\left(\frac{\partial^2 w}{\partial y^2}\right)^2+2\nu\frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{\partial y^2} + 2(1-\nu)\left(\frac{\partial^2 w}{\partial x\partial y}\right)^2\right\}\mathrm{d}x\,\mathrm{d}y. \tag{4}$$

The Rayleigh-Ritz method allows to find an approximate solution to a differential equation with given boundary conditions (Leissa, 1969). It is based on an assumption that the solution can be expressed as a Ritz series:

$$w(x,y,t)=\sum_{i=1}^{M}\eta_i(x,y)q_i(t), \tag{5}$$

where q_i is the generalized displacement and η_i is the i-th Ritz function. The Ritz function needs to satisfy the geometric boundary condition, so for a rectangular plate it is assumed to be a product of the eigenfunctions of a one-dimensional bar u_n:

$$\eta_i(x,y)=u_n(x)u_m(y). \tag{6}$$

The Ritz functions determine, which geometry of the plate will be considered and what boundary condition will be adopted.

Then, the transversal displacement expressed as in (5) is substituted into kinetic and potential energy Eqs. (3) and (4):

$$T=\frac{1}{2}\sum_{i=1}^{M}\sum_{j=1}^{M}\left(\iint_S m_s\eta_i\,\eta_j\,\mathrm{d}x\,\mathrm{d}y\right)\dot{q}_i\dot{q}_j, \tag{7}$$

$$U=\frac{D}{2}\sum_{i=1}^{M}\sum_{j=1}^{M}\left\{\iint_S\left[\frac{\partial^2\eta_i}{\partial x^2}\frac{\partial^2\eta_j}{\partial x^2}+\frac{\partial^2\eta_i}{\partial y^2}\frac{\partial^2\eta_j}{\partial y^2} +2\nu\frac{\partial^2\eta_i}{\partial x^2}\frac{\partial^2\eta_j}{\partial y^2} +2(1-\nu)\frac{\partial^2\eta_i}{\partial x\partial y}\frac{\partial^2\eta_j}{\partial x\partial y}\right]\mathrm{d}x\,\mathrm{d}y\right\}q_i\,q_j. \tag{8}$$

The superimposed dot denotes the time derivative. The kinetic and potential energies can be also written as functions of generalized displacement vector $\mathbf{q}$, mass matrix $\mathbf{M}$ and stiffness matrix $\mathbf{K}$:

$$T=\frac{1}{2}\dot{\mathbf{q}}^{\mathrm{T}}\mathbf{M}\dot{\mathbf{q}}, \tag{9}$$

$$U=\frac{1}{2}\mathbf{q}^{\mathrm{T}}\mathbf{K}\mathbf{q}. \tag{10}$$

The superscript T denotes the transpose. The mass and stiffness matrices, $\mathbf{M}$ and $\mathbf{K}$, depend on Ritz functions, and can be calculated as:

$$M_{ij}=\iint_S m_s\eta_i\eta_j\,\mathrm{d}x\,\mathrm{d}y, \tag{11}$$

$$K_{ij}=D\iint_S\left[\frac{\partial^2\eta_i}{\partial x^2}\frac{\partial^2\eta_j}{\partial x^2}+\frac{\partial^2\eta_i}{\partial y^2}\frac{\partial^2\eta_j}{\partial y^2} +2\nu\frac{\partial^2\eta_i}{\partial x^2}\frac{\partial^2\eta_j}{\partial y^2} +2(1-\nu)\frac{\partial^2\eta_i}{\partial x\partial y}\frac{\partial^2\eta_j}{\partial x\partial y}\right]\mathrm{d}x\,\mathrm{d}y. \tag{12}$$

Finally, the equation of a vibrating structure can be obtained:

$$\mathbf{M}\ddot{\mathbf{q}}+\mathbf{K}\mathbf{q}=\mathbf{Q}, \tag{13}$$

where $\mathbf{Q}$ is the vector of generalized forces. In this paper, electrodynamic actuators are considered. Hence, for actuator positioning purpose, their action can be simplified and taken into account as a force acting on a point:

$$\mathbf{Q} = \iint_S \boldsymbol{\eta} \, f_a \, \mathrm{d}x \, \mathrm{d}y. \tag{14}$$

The harmonic solution of Eq. (13) gives the eigenvector matrix $\mathbf{\Phi}$ and eigenfrequencies ω_i. Replacing $\mathbf{q}$ by $\mathbf{\Phi v}$ and multiplying Eq. (13) on the left by $\mathbf{\Phi}^{\mathrm{T}}$, it gives:

$$\ddot{\mathbf{v}} + \mathrm{diag}(\omega_i^2)\mathbf{v} = \mathbf{\Phi}^{\mathrm{T}}\mathbf{Q}. \tag{15}$$

This equation can be written in a usual state-space form, using the state vector $\mathbf{x}$ truncated at N modes as:

$$\mathbf{x} = [\dot{v}_1, \omega_1 v_1, \dot{v}_2, \omega_2 v_2, \ldots, \dot{v}_N, \omega_N v_N]^{\mathrm{T}}, \tag{16}$$

$$\frac{\partial}{\partial t}\mathbf{x} = \mathbf{Ax} + \mathbf{Bu} \tag{17}$$

with $\mathbf{A} = \mathrm{diag}(\mathbf{A}_i)$, where

$$\mathbf{A}_i = \begin{bmatrix} -2\xi\omega_i & -\omega_i \\ \omega_i & 0 \end{bmatrix}. \tag{18}$$

Damping ratio ξ is determined experimentally. Matrix $\mathbf{B}$ can be expressed as:

$$\mathbf{B} = [b_1, 0, b_2, 0, \ldots, b_N, 0]^{\mathrm{T}}, \tag{19}$$

where b_i is the i-th component of the vector $\mathbf{\Phi}^{\mathrm{T}}\mathbf{Q}$. Matrix $\mathbf{B}$ contains as many columns as the number of actuators.

3. Optimization criterion for actuators locations

The chosen objective function to be minimized expresses control energy required to reach the desired state $\mathbf{x}_{T_1}$ at time $t = T_1$:

$$E = \int_0^{T_1} \mathbf{u}^{\mathrm{T}}(t)\mathbf{u}(t) \, \mathrm{d}t. \tag{20}$$

For the initial state, $\mathbf{x}_0$, the optimal solution requires the following energy transmitted from the actuators to the structure:

$$E_{\mathrm{opt}} = (e^{\mathbf{A}T_1}\mathbf{x}_0 - \mathbf{x}_{T_1})^{\mathrm{T}}\mathbf{W}^{-1}(T_1)(e^{\mathbf{A}T_1}\mathbf{x}_0 - \mathbf{x}_{T_1}), \tag{21}$$

where $\mathbf{W}(T_1)$ is the controllability Gramian matrix defined by:

$$\mathbf{W}(T_1) = \int_0^{T_1} e^{\mathbf{A}t}\mathbf{B}\mathbf{B}^{\mathrm{T}}e^{\mathbf{A}^{\mathrm{T}}t} \, \mathrm{d}t. \tag{22}$$

To minimize control energy with respect to the actuators locations, a measure of the Gramian matrix should be maximized. It has been shown in the literature that instead of using $\mathbf{W}(T_1)$, a steady-state controllability Gramian matrix $\mathbf{W}_c$ can be used for stable systems, when time tends to infinity (Anderson, Moore, 1990):

$$\mathbf{W}_c = \int_0^{\infty} e^{\mathbf{A}t}\mathbf{B}\mathbf{B}^{\mathrm{T}}e^{\mathbf{A}^{\mathrm{T}}t} \, \mathrm{d}t. \tag{23}$$

The steady-state controllability Gramian $\mathbf{W}_c$ can be calculated by solving the Lyapunov equation:

$$\mathbf{A}\mathbf{W}_c + \mathbf{W}_c\mathbf{A}^{\mathrm{T}} + \mathbf{B}\mathbf{B}^{\mathrm{T}} = 0. \tag{24}$$

If the i-th eigenvalue of $\mathbf{W}_c$ corresponding to i-th eigenmode is small, the eigenmode is difficult to control (it can be regulated only if a large control energy is available). To ensure controllability of initial N eigenmodes, the following criterion can be thus considered:

$$J = \min_{i=1,\ldots,N} \lambda_i, \tag{25}$$

where λ_i is the i-th eigenvalue of the steady state controllability Gramian. Such criterion concerns maximization of controllability of the least controllable eigenmode.

As the number of actuators and considered eigenmodes increases, search space size expands and becomes more complex. Hence, memetic algorithms are proposed to solve the optimization problem.

4. Memetic algorithms

Evolutionary algorithms have proven to be a versatile and effective technique for solving nonlinear optimization problems with multiple optima (Goldberg, 1989). Their convergence properties has been discussed in (Greenhalgh, Marshall, 2000). However, they usually require evaluation of numerous solutions resulting in high computational cost. Memetic algorithms are hybrid forms of population-based approach coupled with separate individual learning. Memetic algorithms combine advantages of a global search, like for evolutionary algorithms, and local improvement procedures, which enhance converge to the local optima (Neri *et al.*, 2011). Because of complementary properties, they are particularly useful in solving complex multi-parameter optimization problems, such as the actuators placement.

As shown in Fig. 1, the memetic algorithm starts with a randomly generated population of candidate solutions called individuals. The fitness function is evaluated for each individual. A part of the existing population is selected for further reproduction dependent on the fitness value (individuals fitting better are

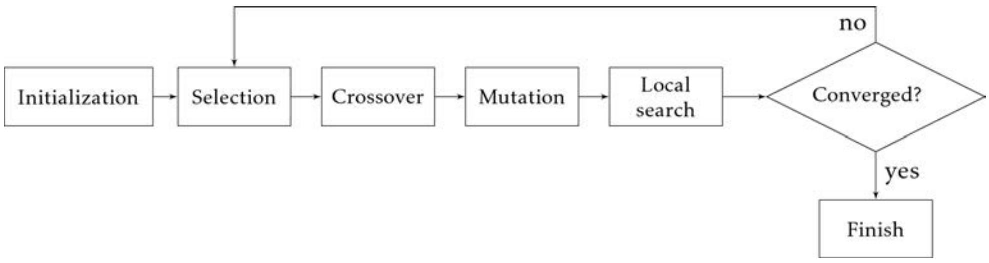


Fig. 1. Memetic algorithm flowchart.

more likely to be chosen). Children solutions are generated by applying one of crossover methods for two or more parents. To maintain genetic diversity, the mutation operator might be used dependent on a predefined probability. Then, a local search technique is employed to improve individual fitness. To maintain a balance between the degree of evolution (exploration) and individual improvement (exploitation), only a portion of the population individuals undergo the learning. Afterwards, a selection is performed, and the process is repeated until a certain termination criterion is met.

The optimization problem considered in this paper, consists of determining the efficient locations of fixed number of actuators. Optimization variables are actuators locations expressed as spatial coordinates. The size of the population is kept the same in each iteration step. Best individuals are kept unchanged in the next generation (elitist selection). The "Hill climbing" technique (Neri *et al.*, 2011) is assumed as the individual learning strategy. The termination criterion is satisfied if no improvement is found in the last m iterations, or the maximum number of iterations is reached.

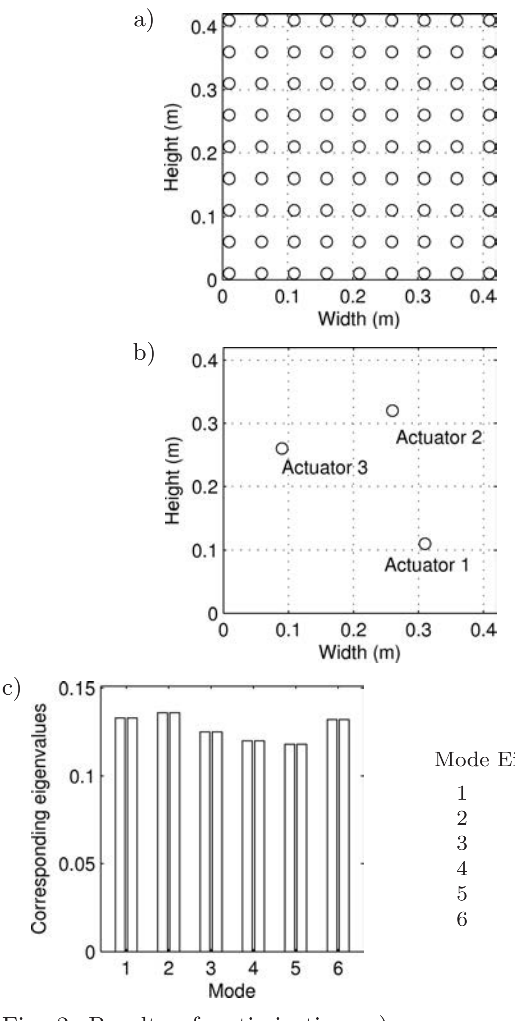
5. Application for a fully-clamped square plate

In this section, application of the proposed method for optimal placement of 3 electrodynamic actuators on a fully-clamped square plate is presented. The objective is to ensure controllability of initial 6 eigenmodes, by maximizing criterion (25). Such assumptions make the analysis sufficiently general to consider both control complexity and application related aspects. Dimensions and characteristics of the plate and actuators are given in Table 1. Obtained eigenvalues of the controllability Gramian corresponding to eigenmodes are presented in Fig. 2c. Actuators locations found are shown in Fig. 2b.

Corresponding shapes and frequencies of the eigenmodes are presented in Fig. 4. Frequency responses of the plate due to excitation by individual actuators with a random signal were measured in 81 uniformly distributed points, depicted in Fig. 2a. The distance between measurement points, and therefore the number

Table 1. Mechanical and electrical properties of the plate and exciters.

Properties	Plate	Exciter EX1
Size [mm]	420x420	⌀70
Thickness [mm]	3	19
Density [Kg/m^3]	2700	–
Mass [Kg]	1.428	0.115
Young modulus [GPa]	70	–
Poisson's ratio	0.35	–
Power handling [W]	–	5



Mode	Eigenvalues
1	0.134
2	0.137
3	0.126
4	0.121
5	0.119
6	0.132

Fig. 2. Results of optimization: a) measurement points, b) actuator locations, c) eigenvalues.

of them, has been adopted to be considerably smaller than the distance between the nodes and anti-nodes of the plate eigenmodes in the frequency band considered. Results averaged over entire plate are presented in Fig. 3. For experimental verification the Polytec laser vibrometer PDV-100 has been used.

a) Actuator 1

b) Actuator 2

c) Actuator 3

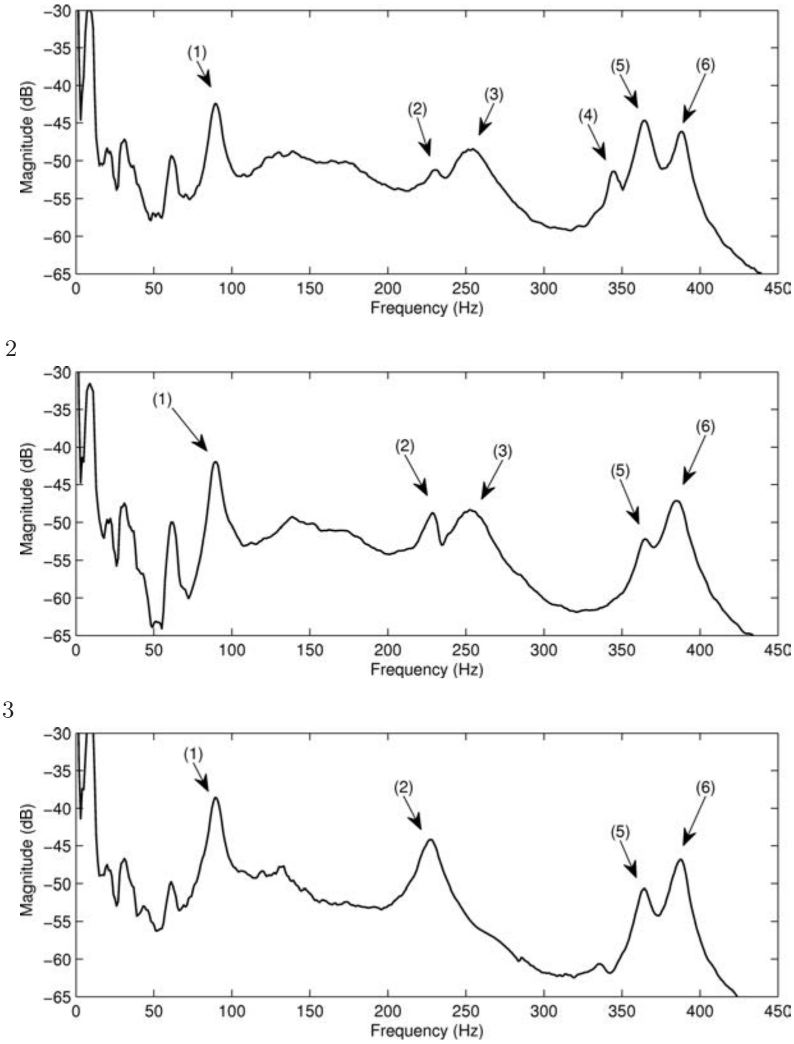


Fig. 3. Magnitudes of surface-averaged frequency responses of the plate due to excitation by individual actuators.

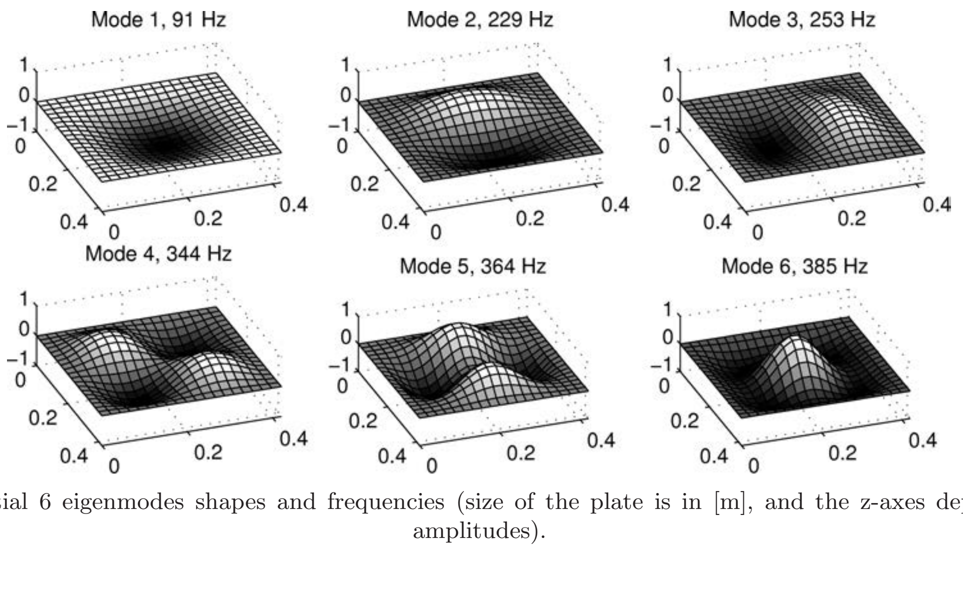


Fig. 4. The initial 6 eigenmodes shapes and frequencies (size of the plate is in [m], and the z-axes depict normalized amplitudes).

Particular eigenmode is considered controllable if the corresponding peak is distinguishable in the frequency response graph. As shown in Fig. 3, individual actuators complement each other. Every actuator excites the first mode, but e.g. the forth mode is well excited only by the actuator 1. Hence, each desired eigenmode is controllable with an acceptable margin.

The results presented above demonstrate in detail the case of 3 actuators and initial 6 eigenmodes to be controlled. However, studies have also been performed for other quantities of actuators and frequency ranges considered. It follows from the analysis that the presented approach is suitable also for a wider frequency band. However, it has been noted that increasing frequency bandwidth for the same number of actuators results in decreased level of controllability, which is possible to achieve. On the other hand, for increasing the number of actuators while maintaining a constant frequency bandwidth, results in increasing the level of controllability. In practical applications usually there are limitations in the number of actuators that can be used. Thus, the frequency band, for which the algorithm is able to find efficient locations is also limited. An excessive extension of the band considered causes that the acquired locations will be a compromise between too many modes to be controlled and, as a result, none of them will be sufficiently controllable.

6. Comparison of evolutionary and memetic algorithms

In this section, performance of evolutionary and memetic algorithms in application to the problem of actuator placement is presented. Due to in-built local search procedures, MA involves more operations than EA for each generation. Extend of the additional computational load depends on adopted parameters and chosen procedures. For the study to be adequate, both algorithms should posses the same computational budget. Therefore, during the test, population in EA consisted of 90 individuals, while MA population had only 20 individuals. Such arrangement resulted in a similar average computation time. Maximum number of generations was set to 30. The probabilities for crossover, mutation and individual learning were 0.7, 0.05 and 0.0 for EA, and 0.5, 0.05 and 0.6 for MA, respectively. It was the best configuration found empirically for the specified problem. Details of the problem specific parameters are described in the previous section.

Both algorithms were started with randomly generated initial population, which affected strongly convergence rate. To obtain statistical measures of their performance, each algorithm was run 100 times. Each particular run is presented in gray in Fig. 5, for distribution of possible results to be visible. The average

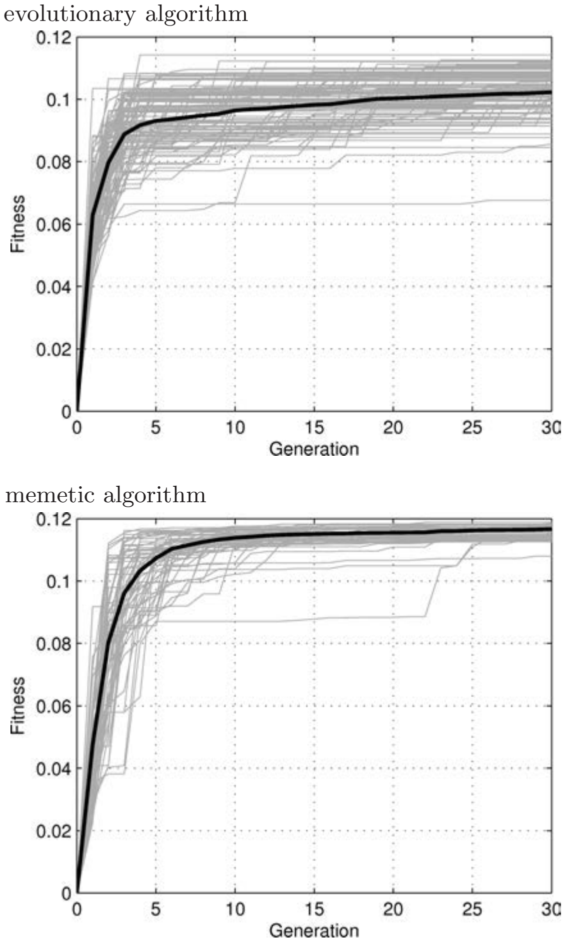


Fig. 5. Multiple runs of optimization algorithms.

result is shown as the bold black line. The summary of the characteristic values is given in Table 2.

Table 2. Comparison of characteristic values.

Properties	Evolutionary algorithm	Memetic algorithm
Runs	100	100
Generations	30	30
Population size	90	20
Crossover probability	0.7	0.5
Mutation probability	0.05	0.05
Individual learning probability	0.0	0.6
Best final fitness	0.114	0.119
Average final fitness	0.102	0.116
Worst final fitness	0.068	0.108

It follows from the analysis that both algorithms are capable of reaching similar level of best value of the fitness function. However, the EA best solution is worse than the MA average solution. This indicates that both of them could be used successfully for solving the optimization problem, but MA provides a better solution. To ensure that obtained solution is near the global optimum, consistency of MA might also be considered as an advantage over EA. Less runs would be necessary in the case of MA, what indicates a better computational efficiency. Additionally, if more complicated structures of multiple plates and with more actuators are considered, benefits of using the MA algorithm shall be more significant (Garg, 2010).

7. Conclusion

A model of a rectangular plate with electrodynamic actuators bonded to its surface has been presented. The Reyleigh-Ritz method has been used to find a solution to a differential equation. Suitability of the model for other geometries and boundary conditions of plates has been pointed out. A controllability-oriented optimization criterion for placing the actuators has been developed, ensuring each mode of the structure to be controllable.

The proposed method has been used to find optimal locations of three electrodynamic actuators on a fully clamped aluminum square plate. Initial six eigenmodes of the plate have been taken into account. Results of an experimental verification confirmed high level of controllability of each mode considered. General features and limitations of the method presented has been outlined.

Performance of evolutionary and memetic algorithms in application to the optimization problem has been compared. The analysis confirmed suitability and efficiency of using memetic algorithms to find the optimal placement of actuators for an active noise-vibration control application. It has been pointed out that benefits of using memetic algorithms shall be more significant if more actuators and more complicated structures is considered.

The plate with correctly distributed actuators is ready to be applied for noise control problems. It is then a multi-output plant, which generally requires a multi-channel control system. However, with a bank of fixed-parameter filters it can be converted to a single output plant, what significantly reduces computational complexity of control systems, as shown in (Mazur, Pawełczyk, 2013b), particularly if the sound-radiating plate exhibits in practice a non-linear character due to improper fixing or properties of the actuators (Mazur, Pawełczyk, 2013a). For active control an LMS-based algorithm can be applied. However, contrary to the problem of electrical noise cancellation or speech enhancement (see, e.g. (Latos, Pawełczyk, 2010)), models of the plant paths including the specific actuators and their physical parameters are then required. These problems are of current interest of the authors.

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