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Constrained controllability of semilinear systems with delayed controls

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Abstract. In the present paper finite-dimensional dynamical control systems described by semilinear ordinary differential state equations with multiple point delays in control are considered. It is generally assumed, that the values of admissible controls are in a convex and closed cone with vertex at zero. Using so-called generalized open mapping theorem, sufficient conditions for constrained local relative controllability near the origin are formulated and proved. Roughly speaking, it will be proved that under suitable assumptions constrained global relative controllability of a linear associated approximated dynamical system implies constrained local relative controllability near the origin of the original semilinear dynamical system. This is generalization to the constrained controllability case some previous results concerning controllability of linear dynamical systems with multiple point delays in the control and with unconstrained controls. Moreover, necessary and sufficient conditions for constrained global relative controllability of an associated linear dynamical system with multiple point delays in control are discussed. Simple numerical example, which illustrates theoretical considerations is also given. Finally, some remarks and comments on the existing results for controllability of nonlinear dynamical systems are also presented.

Key words: controllability, nonlinear control systems, semilinear control systems, constrained controls, delayed control systems.

1. Introduction

Controllability is one of the fundamental concepts in mathematical control theory [1–3]. This is a qualitative property of dynamical control systems and is of particular importance in control theory. Roughly speaking, controllability generally means, that it is possible to steer dynamical control system from an arbitrary initial state to an arbitrary final state using the set of admissible controls. In the literature there are many different definitions of controllability, which strongly depend on class of dynamical control systems and the set of admissible controls [1–3].

In recent years various controllability problems for different types of nonlinear dynamical systems have been considered in many publications and monographs. However, it should be stressed, that the most literature in this direction has been mainly concerned with controllability problems for finite-dimensional nonlinear dynamical systems with unconstrained controls and without delays [1–5] or for linear infinite-dimensional and finite-dimensional dynamical systems with constrained controls and without delays [6–9]. Recently stochastic controllability systems with delays have been considered in the papers [10, 11].

In the present paper, we consider constrained local relative controllability problems for finite-dimensional semilinear dynamical systems with multiple point delays in the control described by ordinary differential state equations. Let us recall, that semilinear dynamical control systems contain linear and pure nonlinear parts in the differential state equations [4, 12]. Moreover, general theory of optimal control problems for dynamical systems with different types of points and distributed delays is presented in monographs [13, 14].

In the sequel, we shall formulate and prove sufficient conditions for constrained local relative controllability in a prescribed time interval for semilinear dynamical systems with multiple point delays in the control with nonlinear term containing delayed controls, which is continuously differentiable near the origin. It is generally assumed that the values of admissible controls are in a given convex and closed cone with vertex at zero, or in a cone with nonempty interior. Proof of the main result is based on a so-called generalized open mapping theorem presented in the paper [15]. Moreover, necessary and sufficient conditions for constrained global relative controllability of an associated linear dynamical system with multiple point delays in control are recalled and discussed. Simple numerical example, which illustrates theoretical considerations is also given. Finally, some remarks and comments on the existing results for controllability of nonlinear dynamical systems are also presented.

Roughly speaking, it will be proved that under suitable assumptions constrained global relative controllability of a linear associated approximated dynamical system with delays implies constrained local relative controllability near the origin of the original semilinear dynamical system with delays. It should be pointed out that the results of this paper are generalizations to more complicated semilinear systems constrained controllability conditions, previously presented in papers [9, 16–19] for simpler semilinear systems with delays.

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2. System description

In this paper we study the semilinear control system with multiple point delays in the control described by the following ordinary differential state equation

$$\dot{x}(t) = Cx(t) + F(x(t), u(t), u(t-h_1), ..., u(t-h_j), ..., u(t-h_M)) + \sum_{j=0}^{j=M} D_j u(t-h_j) \quad \text{for} \quad t \in [0,T], \quad T > h, \tag{1}$$

with zero initial conditions:

$$x(0) = 0, \quad u(t) = 0 \quad \text{for} \quad t \in [-h, 0), \tag{2}$$

where the state $x(t) \in R^n$ and the control $u(t) \in R^m$, C is $n \times n$-dimensional constant matrix, D_j, $j = 0, 1, 2, ..., M$ are $n \times m$ dimensional constant matrices, $0 = h_0 < h_1 < ... < h_j < ... < h_M = h$ are constant delays.

Moreover, let us assume that the nonlinear mapping $F : R^n \times R^m \times R^m \ldots \times R^m \to R^n$ is continuously differentiable near the origin in the space $R^n \times R^m \times R^m \ldots \times R^m$ and such that $F(0, 0, 0, \ldots, 0) = 0$.

In practice admissible controls are always required to satisfy certain additional constraints. Generally, for arbitrary control constraints it is rather very difficult to give easily computable criteria for constrained controllability even for dynamical systems without delays. However, for some special cases of the conical constraints it is possible to formulate and prove simple algebraic constrained controllability conditions. Therefore, in the sequel we shall assume that the set of values of admissible controls $U_c \subset R^m$ is a given closed and convex cone with nonempty interior and vertex at zero. Thus the set of admissible controls U_{ad} for the dynamical control system (1) has the following form $U_{ad} = L_\infty([0,T], U_c)$.

Then for a given admissible control $u(t)$ there exists a unique solution $x(t;u)$ for $t \in [0,T]$, of the differential state Eq. (1) with zero initial condition (2) described by the integral formula [6, 18].

$$x(t;u) = \int_0^t S(t-s)((F(x(s), u(s), u(s-h_1), ..., u(s-h_j), ..., u(s-h_{M)}) + \sum_{j=0}^{j=M} D_j u(t-h_j))ds \tag{3}$$

where $S(t) = \exp(Ct)$ is $n \times n$-dimensional transition matrix for the linear part of the semilinear control system (1).

Remark 1. Since in special case matrices A and D_j, $j = 0, 1, 2, \ldots, M$ may be zero matrices, then dynamical system (2.1) represents also mathematical model of general nonlinear dynamical systems.

For the semilinear dynamical system with multiple point delays in the control (1), it is possible to define many different concepts of controllability. In the sequel we shall focus our attention on the so-called constrained relative controllability in the time interval $[0,T]$. In order to do that, first of all let us introduce the notion of the attainable set at time $T > 0$ from zero initial conditions (2), denoted by $K_T(U_c)$ and defined as follows [2, 3, 17].

$$K_T(U_c) = \{x \in X : x = x(T,u), u(t) \in U_c \ \text{for a.e.} \quad t \in [0,T]\} \tag{4}$$

where $x(t,u)$, $t > 0$ is the unique solution of the differential Eq. (1) with zero initial conditions (2) and a given admissible control u. It should be pointed out, that under the assumptions stated on the nonlinear term F such solution always exists [6, 18].

Now, using the concept of the attainable set given by the relation (4), let us recall the well known (see e.g. [2, 3, 14] definitions of constrained relative controllability in $[0,T]$ for dynamical system (1).

Definition 1. Dynamical system (1) is said to be U_c-locally relative controllable in $[0,T]$ if the attainable set $K_T(U_c)$ contains a certain neighborhood of zero in the space R^n.

Definition 2. Dynamical system (1) is said to be U_c-globally relative controllable in $[0,T]$ if $K_T(U_c) = R^n$.

Remark 2. Finally, it should be stressed, that in a quite similar way, we may define constrained global absolute controllability and local absolute controllability for delayed dynamical system (1) [2]. However, since in this case the state space is in fact infinite-dimensional function space, then it is necessary to distinguish between exact absolute controllability and approximate absolute controllability. Absolute controllability of delayed dynamical system (1) is not considered in this paper.

3. Preliminaries

In this section, we shall introduce certain notations and present some important facts taken directly from the general theory of nonlinear operators in Banach spaces.

Let U and X be given Banach spaces and $g(u) : U \to X$ be a nonlinear mapping continuously differentiable near the origin 0 of the space U. Let us suppose for convenience that $g(0) = 0$. It is well known from the so-called implicit-function theorem (see e.g. [15]) that, if the Frechet derivative at zero, which is a bounded linear operator $Dg(0) : U \to X$ maps the space U onto the whole space X, then the nonlinear map g transforms a neighborhood of zero in the space U onto some neighborhood of zero in the space X.

Now, let us consider the more general case when the domain of the nonlinear operator g is Ω, an open subset of U containing 0. Let U_c denote a closed and convex cone in the space U with vertex at 0.

In the sequel, we shall use for controllability investigations some property of the nonlinear mapping g, which is a consequence of a generalized open-mapping theorem [15]. This result seems to be widely known, but for the sake of completeness we shall present it here, though without proof and in a slightly less general form sufficient for our purpose.

Lemma 1. [15] Let X, U, U_c, and Ω be as described above. Let $g : \Omega \to X$ be a nonlinear mapping and suppose that on Ω nonlinear mapping g has Frechet derivative Dg, which is continuous at 0. Moreover, suppose that $g(0) = 0$ and assume that linear map $Dg(0)$ maps U_c onto the whole space X. Then there exist neighborhoods $N_0 \subset X$ about $0 \in X$ and $M_0 \subset \Omega$ about $0 \in U$ such that the nonlinear equation $x = g(u)$ has, for each $x \in N_0$, at least one solution $u \in M_0 \cap U_c$, where $M_0 \cap U_c$ is a so called conical neighborhood of zero in the space U.

4. Controllability conditions

In this section we shall study constrained local relative controllability near the origin in the time interval $[0,T]$ for semilinear dynamical system (1) using the associated linear dynamical system with multiple point delays in the control

$$\dot{z}(t) = Az(t) + \sum_{j=0}^{j=M} B_j u(t-h_j) \quad \text{for} \quad t \in [0,T], \tag{5}$$

with zero initial condition $z(0) = 0$, $u(t) = 0$, for $t \in [-h,0)$ where constant matrices

$$A = C + D_x F(0,0,\ldots,0) \quad \text{and} \quad B_j = D_j + D_{u(t-h_j)}F(0,0,\ldots,0) \quad \text{for} \quad j = 0,1,\ldots,M. \tag{6}$$

The main result of the paper is the following sufficient condition for constrained local relative controllability of the semilinear dynamical system with multiple constant delays in the control (1).

Theorem 1. Suppose that
(i) $F(0,0,\ldots,0) = 0$,
(ii) $U_c \subset R^m$ is a closed and convex cone with vertex at zero,
(iii) The associated linear control system with multiple point delays in the control (5) is U_c-globally relative controllable in $[0,T]$.

Then the semilinear dynamical control system with multiple point delays in the control (1) is U_c-locally relative controllable in $[0,T]$.

Proof. Let us define for the nonlinear dynamical system (1) a nonlinear map $g : L_\infty([0,T],U_c) \to R^n$ by $g(u) = x(T,u)$.

Similarly, for the associated linear dynamical system (5), we define a linear map $H : L_\infty([0,T],U_c) \to R^n$ by $Hv = z(T,v)$.

By the assumption (iii) the linear dynamical system (5) is U_c-globally relative controllable in $[0,T]$. Therefore, by the Definition 2 the linear operator H is subjective i.e., it maps the cone U_{ad} onto the whole space R^n. Furthermore, by Lemma 1 we have that $Dg(0) = H$.

Since U_c is a closed and convex cone, then the set of admissible controls $U_{ad} = L_\infty([0,T],U_c)$ is also a closed and convex cone in the function space $L_\infty([0,T],U)$. Therefore, the nonlinear map g satisfies all the assumptions of the generalized open mapping theorem stated in the Lemma 1. Hence, the nonlinear map g transforms a conical neighborhood of zero in the set of admissible controls U_{ad} onto some neighborhood of zero in the state space R^n. However, this is by Definition 1 equivalent to the U_c-local relative controllability in $[0,T]$ of the semilinear dynamical control system (1). Hence, our theorem follows.

Let us observe, that in practical applications of the Theorem 1, the most difficult problem is to verify the assumption (iii) about constrained global controllability in a given time interval of the linear dynamical system with multiple point delays in the control (5) [1–3, 8–9]. In order to avoid this serious disadvantage, we may use the Theorems and Corollaries given in the next section.

5. Constrained controllability conditions for linear systems

In this section for completeness of considerations we shall recall well known necessary and sufficient conditions for constrained relative controllability of time-invarinat linear dynamical systems with constant multiple point delays in the conically constrained control.

First of all, for simplicity of notation, let us denote:

$$\tilde{B}_k(t) = \begin{bmatrix} B_0 & B_1 & \ldots & B_j & \ldots & B_k \end{bmatrix}$$

where $\tilde{B}_k(t)$ are $n \times m(k+1)$-dimensional constant matrices defined for $h_k < t \leq h_{k+1}$, $k = 0,1,2,...,M$, where $h_{M+1} = +\infty$.

The main result is the following necessary and sufficient condition for constrained relative controllability of the linear dynamical system (5).

Theorem 2. Linear control system with multiple point delays in the control (5) is U_c-relative controllable in $[0,T]$ for $h_k < T \leq h_{k+1}$, $k = 0,1,2,...,M$, $h_{M+1} = +\infty$, if and only if the linear dynamical control system without point delays in the control

$$\dot{x}(t) = Ax(t) + \tilde{B}_k(t)v(t), \tag{7}$$

is V_c-controllable in $[0,T]$, where $v(t) \in V_{ad} = L_\infty([0,T], V_c)$, and $V_c = U_c \times U_c \times \ldots \times U_c \in R^{m(k+1)}$ is a given closed and convex cone with nonempty interior and vertex at zero.

Proof. First of all, taking into account zero initial conditions (2) and changing the order of integration, let us transform equality (3) as follows

$$x(t;u) = \int_0^t \exp(A(t-s)) \sum_{j=0}^{j=M} B_j u(s-h_j)ds =$$
$$= \sum_{j=0}^{j=M} \int_0^t \exp(A(t-s))B_j u(s-h_j)ds =$$
$$= \sum_{j=0}^{j=k} \int_0^{t-h_j} \exp(A(t-s+h_j))B_j u(s)ds$$

for t satisfying inequalities $h_j < t \leq h_{j+1}$, $j = 0,1,2,...,k-1$.

Let us observe that since the matrices $\exp(A(t-s+h_j)$ are always nonsingular therefore, they do not change controllability property of dynamical system. Hence, relative controllability of linear system with delays in control (5) is in fact equivalent to controllability of the following linear system without delays in the control (7) [2]. Hence Theorem 2 follows.

Now, using results concerning constrained controllability of linear system without delays (7) (see e.g. [1–3] for more details) we shall formulate and prove necessary and sufficient conditions for constrained relative controllability of linear dynamical systems with delays in control (1).

Remark 3. The monographs [2, 13, 14] contain many quite general models of time-varying linear finite-dimensional dynamical systems both with distributed and time-varying multiple point delays in the control.

Theorem 3. Suppose the set U_c is a cone with vertex at zero and a nonempty interior in the space R^m. Then the linear dynamical control system (5) is U_c-relatively controllable in $[0,T]$, $h_k < T \le h_{k+1}$, $k = 0,1,2,\dots,M$ if and only if
(1) it is relative controllable in $[0,T]$ without any constraints, i.e.

$$\text{rank}[B_0, B_1, ..., B_{k.}, AB_0, AB_1, ..., AB_k, A^2B_0, A^2B_1, ..., A^2B_k, ..., A^{n-1}B_0, A^{n-1}B_1, ..., A^{n-1}B_k] = n,$$

(2) there is no real eigenvector $w \in R^n$ of the transpose matrix A^T satisfying

$$w^T\tilde{B}_k v \le 0 \quad \text{for all} \quad v \in V_c.$$

Proof. First of all let us recall, that by Theorem 2 constrained relative controllability of linear systems with delays in control given by linear state equation (5) is equivalent to constrained controllability of linear system without delays (7). On the other site, it is well known [1–3] that necessary and sufficient conditions for constrained controllability of system (7) are exactly conditions (1) and (2) given above. Hence our Theorem 3 follows.

Let us observe, that for a special case when the final time $T < h_1$, controllability problem in time interval $[0,T]$ may be reduced to the controllability problem for dynamical system without delays in the control of the following form [2].

$$\dot{x}(t) = Ax(t) + B_0u(t). \tag{8}$$

Therefore, using standard controllability conditions given in [4] or [5] we can formulate the following Corollary.

Corollary 1. [2, 3]. Suppose that $T < h_1$ and the assumptions of Theorem 3 are satisfied. Then the linear dynamical control system (.1) is U_c-controllable in $[0,T]$ if and only if it is controllable without any constraints, i.e.

$$\text{rank}[B_0, AB_0, A^2B_0, ..., A^{n-1}B_0] = n,$$

and there is no real eigenvector $w \in R^n$ of the matrix A^T satisfying $w^TB_0u \le 0$ for all $u \in U_c$.

Moreover, for the linear dynamical control system (5), with matrix A having only complex eigenvalues Theorem 5.2 reduces to the following Corollary.

Corollary 2. [2, 3]. Suppose that matrix A has only complex eigenvalues. Then the linear dynamical control system (5) is U_c-relative controllable in $[0,T]$, $h_k < T \le h_{k+1}$, $k = 0,1,2,\dots,M$ if and only if it is relative controllable without any constraints.

It should be pointed out, that for scalar admissible controls, i.e. for $m = 1$, and $U_c = R^+$, the condition (2) of Theorem 3.2 may holds when the matrix A has only complex eigenvalues. Hence we have the following Corollary.

Corollary 3. [2, 3]. Suppose that $m = 1$. Then the linear dynamical control system (1) is U_c-relative controllable in $[0,T]$, $h < T$, if and only if it is relative controllable in $[0,T]$ without any constraints, and matrix A has only complex eigenvalues.

Finally, let us consider the simplest case, when the final time T is small enough, i.e. $T \le h_1$, and the matrix A has only complex eigenvalues. In this case relative constrained controllability problem in $[0,T]$ for delayed dynamical system reduces to a very well known in the literature controllability problem for dynamical system without delays. In this case we have matrix rank controllability condition [2, 3]. given in Corollary 4.

Corollary 4. [2, 3]. Suppose that $T \le h_1$, $U_c = R^+$ and matrix A has only complex eigenvalues. Then the linear dynamical control system (4.1) is U_c-controllable in $[0,T]$ if and only if it is controllable without any constraints, i.e.

$$\text{rank}[B_0, AB_0, A^2B_0, ..., A^{n-1}B_0] = n.$$

Remark 4. From the above theorems and corollaries directly follows, that for delayed dynamical systems constrained relative controllability strongly depends on the delays and on the length of the time interval $[0,T]$.

6. Example

Let us consider the following simple illustrative example. Let the semilinear finite-dimensional dynamical control system with two point constant delays in control is defined on a given time interval $[0,T]$, $T > h = h_2$, and has the following form

$$\begin{aligned} \dot{x}_1(t) &= -x_2(t) + u(t-h_1) \\ \dot{x}_2(t) &= \sin x_1(t) - \cos u(t-h_2) + 1 \end{aligned}. \tag{9}$$

Therfeore, $n = 2$, $m = 1$, $M = 2$, $0 = h_0 < h_1 < h_2 = h$, $x(t) = (x_1(t), x_2(t))^T \in R^2 = X$, $U = R$, and using the notations given in the previous sections matrices A and B and the nonlinear mapping F have the following form

$$C = \begin{bmatrix} 0 & -1 \\ 0 & 0 \end{bmatrix}, \quad D_0 = \begin{bmatrix} 0 \\ 0 \end{bmatrix},$$

$$D_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad D_2 = \begin{bmatrix} 0 \\ 0 \end{bmatrix},$$

$$F(x_1(t), x_2(t), u(t), u(t-h_1), u(t-h_2)) = \begin{bmatrix} 0 \\ \sin x_1(t) - \cos u(t-h_2) + 1 \end{bmatrix}.$$

Moreover, let the cone of values of controls $U_c = R^+$, and the set of admissible controls $U_{ad} = L_\infty([0,T], R^+)$. Hence, we have

$$F(0,0,0,0,0) = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \quad D_xF(x) = \begin{bmatrix} 0 & 0 \\ \cos x_1 & 0 \end{bmatrix},$$

$$D_xF(0,0,0,0,0) = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix},$$

$$A = C + D_xF(0,0,0,0,0) = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix},$$

$$B_0 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \quad B_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}.$$

Therefore, the matrix C has only complex eigenvalues and

$$\text{rank}\,[B_0, B_1, B_2, AB_0, AB_1, AB_2] = \text{rank}\begin{bmatrix} 0 & 1 & 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix} = 2 = n.$$

Hence, both assumptions of the Corollary 4.2 are satisfied and therefore, the associated linear dynamical control system (5) is R^+-globally controllable in a given time interval $[0,T]$. Then, all the assumptions stated in the Theorem 1 are also satisfied and thus the semilinear dynamical control systems (7) is R^+-locally controllable in $[0,T]$. However, it should be mentioned, that

$$\text{rank}\,[B_0, B_1, AB_0, AB_1] = \text{rank}\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} = 1 < n = 2.$$

Moreover, since $(\sin x_1(t) + 1) \geq 0$, therefore, the semilinear dynamical control system (9) is not relative controllable in the interval $[0,T]$, for $T < h_2$, even for unconstrained controls.

7. Concluding remarks

In the paper sufficient conditions for constrained local relative controllability near the origin for semilinear finite-dimensional dynamical control systems with multiple point delays in the control have been formulated and proved. In the proof of the main result generalized open mapping theorem [15] has been used. These conditions extend to the case of constrained relative controllability of finite-dimensional semilinear dynamical control systems the results published in [14,16,17,19] for simpler semilinear control systems.

The method presented in the paper is quite general and covers wide class of semilinear dynamical control systems. Therefore, similar constrained controllability results may be derived for more general class of semilinear dynamical control systems. For example, it seams, that is possible to extend sufficient constrained controllability conditions given in the previous section for semilinear dynamical control systems with distributed delay in the control or with point delays in the state variables and for the discrete-time semilinear control systems.

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